TOPOGRAPHER and ASSISTANT,
Showing mode of carrying instruments.

PHOTOGRAPHIC SURVEYING

INCLUDING THE ELEMENTS OF

DESCRIPTIVE GEOMETRY

AND

PERSPECTIVE

BY

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PREFACE

The first edition of this book was issued in 1889. It was prepared solely for the use of the surveyors employed by the Department of the Interior on photographic surveys; the edition of fifty copies was lithographed in the surveys office. Some copies found their way outside, and, owing to the favourable comments which they received, the whole edition was exhausted in less than a year. Photographic surveys have since been undertaken for many purposes which were not contemplated at their inception, and a new edition of the book has become necessary for the guidance of the surveyors engaged in the work.

The photographic method is known by many names : photogrammetry, metrophotography, topophotography, iconometry, etc.

It seems that iconometry is the most appropriate name, because it expresses the principle of the method, which is to measure by means of perspectives. Whether the perspectives are photographs, or whether they are produced otherwise, is immaterial so far as the method is concerned. Neither this term nor any other has yet been generally adopted.

The conception of this method is due to Laussedat: his first experiments were made in 1849, the perspectives being drawn with a camera lucida. His paper on the subject, written in 1850, was published in 1854. Shortly after, he substituted photography for the camera lucida. He gave a full exposition of the method in various papers, and his work was so complete that little has been added to it since. Wherever photographic surveys are now made, they are executed by the application of the principles laid down by Laussedat.

In Germany, Meydenbauer was the first to give his attention to the new method. His investigations, commenced in 1865 or thereabouts, were continued by Jordan, Hauck, Koppe and many others.

In Italy, the celebrated engineer Porro, who was acquainted with Laussedat and knew his work, proposed for surveying purposes, a

camera built to receive spherical plates. The idea was not practical and was never carried out. In 1875 and 1876, some experiments were made by Lieutenant Manzi Michele, but they were not favourably received. To Major General Annibale Ferrero, Director of the Geographical Military Institute, is due the credit of initiating the ordnance photographic surveys of the present day. Their execution was entrusted to engineer L. Pio Paganini with a staff of able assistants. The work of the institute is very remarkable and deserves careful study.

In Austria, the first important surveys were made by Mr. Vincenz Pollack, chief engineer of the state railways. Some beautiful mountain maps have lately been printed by Giesecke and Devrient, of Leipzig, for the German and Austrian Alpine club. They are the work of S. Simon. He made use of the original ordnance survey, filling in the topographical details from photographs. These maps represent one of the most successful applications of photographic surveying. Schiffner's and Steiner's writings on the subject are among the best, and the Austrian instrument makers, such as R. Lechner or Starke and Kammerer, have surveying cameras of many patterns to select from.

It is in Canada that the method has received its most extensive application; it was first employed when the surveys of Dominion lands were extended to the Rocky Mountains. In the prairies, our operations are limited to defining the boundaries of townships and sections; these lines form a network over the land by means of which the topographical features, always scarce in the prairies, are sufficiently well located for general purposes.

In passing to the mountains, the conditions are entirely different. The topographical features are well marked and numerous; the survey of the section lines is always difficult, often impossible, and in most cases useless. The proper administration of the country required a tolerably accurate map, and means had to be found to execute it rapidly and at a moderate cost. The ordinary methods of topographical surveying were too slow and expensive for the purpose; rapid surveys, based on triangulations and sketches, were tried and proved ineffectual; then photography was resorted to.

Up to 1892, the photographic surveys were confined to the Rocky Mountains, in the vicinity of the Canadian Pacific Railway; at the

end of that year, they covered about 2,000 square miles. In the same year, an International Boundary Commission was appointed to examine the country along the boundary between Canada and the United States Territory of Alaska. The Canadian Commissioner, Mr. W. F. King, decided to carry out his share of the work by photography. In 1893 and 1894, his parties surveyed about 14,000 square miles.

Irrigation surveys were commenced last year in the south-westerly part of the North-west Territories, where the rainfall is not quite sufficient for agricultural purposes. In addition to the gauging of streams, the establishment of bench marks, etc., it is necessary to ascertain the catchment areas and to define the sites best adapted for reservoirs. For this purpose photography has again been resorted to in the foot hills and on the eastern slope of the mountains. It has, in this case, a peculiar advantage. Whether or not a site is a favourable one for a reservoir cannot be known until the plan has been partly plotted. It must be possible to bring water to the proposed place and to run it off; the capacity must also be adequate. If favourable, a detailed survey of the site is required. With the ordinary surveying instruments, a preliminary survey has to be made; if, after plotting it, the site is found favourable, the topographer has to go over the ground a second time to make a detailed survey. Or, the whole of the work may be executed at once, with the contingency that the detailed survey may turn out useless. With the camera, the plan may be plotted so far, and so far only, as required; the photographs which furnish a general plan, can be made to give all the detail wanted without going again into the field. Whether the site is a good one or not, there is no labour wasted.

Notwithstanding the many publications on photographic surveying, the great advantages assigned to it and the numerous experimental surveys executed, it has not yet come into general use; in many quarters there is still an adverse feeling against it. There is such a fascinating simplicity about the method that it is at first difficult to understand the reasons which prevent its adoption. Can anything more convenient be conceived than a method which enables a topographer to gather rapidly on the ground the material for his maps and to construct them afterwards at leisure in his office? In the first edition of this book, I endeavoured to explain this anomaly. The large scales of

European surveys were given as one of the reasons. Col. Laussedat took very strong exception to this explanation, contending that photography can be used to advantage whenever views can be had covering a large space of ground. His contention is no doubt correct; but the advantages are not so great then as with small scales. This can be illustrated by comparing a plane-table-stadia survey in a dry climate with a photographic survey. When the scale is so large, and the points fixed are so close together, that the topographer takes as much time to plot one point as the rodman takes to move from this point to the next one, then there is not a very great difference in the cost of the two surveys. The advantage of the photographic method is that the plotting being done in the office, the field expenditure of the topographer and the cost of his party during the construction of the map, are dispensed with. Against this, there is the disadvantage that the plotting by intersections is more laborious than the plane table plotting by directions and distances. It may be that the camera still has the advantage, but not to such an extent as with a small scale survey. All this, of course, rests on the assumption that the climate permits of outside work every day with the plane table.

The experience of eight years has modified my views on the causes which have prevented the adoption of the method. I am now inclined to believe that they are simply to be found in the failure of those who tried it. I soon discovered that the apparent simplicity of photographic surveying is a delusion, and, that under no circumstances has a topographer to display more skill and ability than when using the camera. He requires not only experience, but a combination of the faculties which make an accomplished topographer. Unlike other methods, it presents nothing before his eyes to show the progress of the work or the gaps that may exist in it. His undeveloped plates are his only records. Every time he exposes one, he must have present in his mind what it will give, what amount of information he can extract from it, what constructions he will apply, what further views are necessary and how they will combine. These acquirements are not the lot of every topographer, and unless a man is well qualified, his attempts at photographic surveying cannot be successful. All this, however, is only the beginning of the surveyor's troubles; it is in the purely photographic part of the work that he finds the most frequent cause of failure. The kind of views wanted, if they include distance, are the

most difficult to obtain. They cannot be made on ordinary plates; orthochromatic plates, although suitable, present difficulties of their own. The prints made from the negatives are another source of trouble; if printed direct on thin albumen paper, they are distorted in washing, and unless a cumbrous camera be employed, they are too small for accurate plotting. Enlargements on bromide paper will, nine times out of ten, be of the soot and chalk kind, without any half-tones. The production of these enlargements in large numbers, and of uniform excellence, was almost impossible before we were taught by Messrs. Hurter and Driffield the relation between photographic negatives and their positives.

Taking all this into consideration, what chance is there that a surveyor, at a first trial, and generally with an extemporized apparatus, will be successful? A failure is almost inevitable, and it is quite natural that he should discard the method as an unreliable one.

There is another difficulty which I would mention with diffidence if I had not as good an authority on my side as Colonel Laussedat. He was the first to elaborate this method, and still seems to be the one who has the clearest conception of its essential principles. I refer to the refined phototheodolites, which appear to be the favourite European instruments. Their object is to secure great precision, the negatives being measured, often under a microscope, and these measurements submitted to calculation or to some other elaborate process. To apply photography in this way is to misunderstand the function of the camera, which is to replace the plane table, and to sacrifice its main advantages. Any degree of precision may be attained, but it must be done, as with the plane table, by the multiplication of stations and views and not by the employment of logarithmic tables.

I hope to show in these pages how the difficulties assigned as causes of failure can be overcome: if they are not, the method is not to be blamed, but the men who apply it. Properly used, it gives results far beyond what can be accomplished by any other process.

The plane table is accepted generally, although not universally, as the instrument of the topographer. Let us compare the cost of plane table and camera surveys, assuming that the plotting is made by intersections in both cases.

In the climate of the Rocky Mountains, where the work under my direction has been executed, one half of the number of days during a season is lost through smoke, fog, rain and snow storms, and our experience is that it takes three days in the office to plot the work of one day in the field. Thus for every day's work in the field, there is another day lost, on account of the weather and three days spent in the office, or five days altogether.

Assuming that the plane table can be used in the field whenever the weather is fair enough for the camera, which is not the case, also that the topographer can plot and draw in the field as quickly as in the office, where he has every convenience at hand, the same survey by the plane table would require the same length of time at actual work or four days. To this we must add four days lost on account of the weather, or eight days altogether.

The cost of our parties in the field is $20.50 per diem: at office work the only expense is the salary of the topographer, $5.00 per diem. Summing up we find the comparative cost as follows:

PLANE TABLE.

8 days in the field, at $20.50 per diem..........	$164

CAMERA.

2 days in the field, at $20.50 per diem..........	$41
3 " office, at 5.00 "	15
	$56

This shows that the plane table survey would cost at least three times as much as the camera survey. In reality the difference is greater, because part of the work, as well on the ground as in the office, is executed by the assistant, an arrangement which cannot very well be made with the plane table. The figures above are derived from our practice; with more views or more detailed plotting, the difference in cost would be still more in favour of the camera.

If we analyse the causes of the superiority of the camera, we find that a very small portion of the topographer's time is spent in surveying operations. Nearly the whole of it is devoted to travelling for

the purpose of seeing the country and he can map all that he, or rather his camera, can see. His work consists of two distinct parts ; on the ground, he simply collects data, and, with the exception of a few angles, does not waste any of his time in plotting or making measurements. This is left for the office, where the only expenses are the salaries of the surveyor and assistant. In the next place, the party consisting of an assistant and two men is, if not smaller, at least as small and inexpensive as for any other kind of survey. One man is sufficient to carry our camera and tripod almost anywhere, while an ordinary plane table, if it could be taken where our camera has been, could not be carried there by any single man.

It is objected that plotting from photographs is more laborious than plotting on the plane table. There is indeed a slight additional labour ; against this may be set off the fact that no useless line is ever drawn, as when, on the plane table, a point is sighted upon which cannot be recognized from the next station. The greater convenience of working in an office, instead of in the open air, turns the scale in favour of the camera. But photography has an overwhelming advantage in the numerous processes which the laws of perspective place at the disposal of the topographer. The plane table cannot compete with the perspectometer or the perspectograph.

Another objection is, that points cannot be so easily identified on photographs, nor the forms of the surface so truly represented, as when the topographer has the ground under his eyes. This is a mistaken idea; there is no difficulty whatever in identifying any number of points on moderately good photographs, and, moreover, the topographer does not need, as with the plane table, to trust to his memory in order to recognize them. The undulations of the ground are, it is true, less distinct on the photographs, but this is more than compensated by the advantage of having, side by side, views of the same place from several stations.

The practice of photographic surveying requires a thorough knowledge of descriptive geometry and perspective. These sciences are not included in the programme of examination of Dominion Land Surveyors, and, as it is for them that this book is prepared, I deem it proper to explain the principles of both in a concise form, keeping in view the special purpose of their application to Perspective Surveying.

I have reproduced, as nearly in their own words as possible, the remarkable investigations of Messrs. Hurter and Driffield; they represent the most important advance in photography during recent years, and may be given as a model of accurate scientific research. An intimate understanding of their work is indispensable to the photographic surveyor.

A few explanations are given on the principles and mode of action of photographic lenses; the description of anastigmats and of their peculiarities is from a lecture by their inventor, Dr. Paul Rudolf.

Considerable space is devoted to perspective instruments; as now constructed, they are almost useless for purposes of topographical surveying, but, there is no reason why they should not be made sufficiently precise, all that is required being more perfect workmanship. For architectural surveys, they may prove most useful.

A short reference is made to secret and balloon surveying. Of course the subject is of no practical interest to Canadian surveyors, but it is well that those engaged on photographic surveys should have some knowledge of everything pertaining to photographic surveying.

The Canadian Surveys are by far the most extensive that have ever been attempted. For this reason, if for no other, it is hoped this account of their mode of execution will prove acceptable to those interested in the development of the science of Surveying.

E. DEVILLE.

LIST OF THE MOST IMPORTANT PUBLICATIONS ON PHOTOGRAPHIC SURVEYING.

1854. *Laussedat.*—Mémoire sur l'emploi de la chambre claire dans les reconnaissances topographiques. Mémorial de l'officier du génie, N° 16, 1854.

1859. *Laussedat.*—Analyse d'un mémoire sur l'emploi de la photographie dans le levé des plans. Comptes rendus de l'Académie des Sciences, 1859.

1860. *Laugier et Daussy.*—Rapport sur le mémoire de M. Laussedat. Comptes rendus de l'Académie des Sciences, 1860.

1862. *Paté.*—Application de la photographie à la topographie militaire.

1864. *Laussedat.*—Mémoire sur l'emploi de la photographie dans le levé des plans. Mémorial de l'officier du génie. No. 17, 1864.

1865. *Girard.*—Laussedat's Arbeiten in Bezug auf die Anwendung der Photographie zur Aufnahme von Planen. Photographisches Archiv.

1866. *Jouart.*—Application de la photographie aux levés militaires.

1867. *Meydenbauer.*—Uber die Anwendung der Photographie zu Architektur und Terrain aufnahmen. Zeitschrift fur Bauwesen.

1874. *Javary.*—Mémoire sur l'application de la photographie aux arts militaires. Mémorial de l'officier du génie. No. 22, 1874.

1876. *Jordan.*—Verwerthung der Photographie zu geometrischen Aufnahmen. Zeitschrift fur Vermessungskunde.

1883. *Hauck.*—Neue Construction der Perspective und Photogrammetrie. Theorie der trilinearen Verwandtschaft ebener Systeme. Journal fur reine und angewandte Mathematik.

1884. *Hauck.*—Mein perspectivischer Apparat. Festschrift der K. techn. Hochschule in Berlin.

1886. *Bornecque.*—La photographie appliquée au lever des plans.

1887. *Stolze.*—Photogrammetrie. Stein's "Das Licht im Dienste wissenschaftlicher Forschung.

1888. *Reed.*—Photography applied to surveying.

1889. *Paganini.*—La fototopografia in Italia.

1889. *Le Bon.*—Les levers photographiques et la photographie en voyage.

1889. *Koppe.*—Die Photogrammetrie oder Bildmesskunst.

1889. *Moessard.*—Le cylindrographe, appareil panoramique.

1891-93. *Steiner.*—Die Photographie im Dienste des Ingenieurs.

1892. *Schiffner.*—Die Photographische Messkunst.

1892. *Legros.*—Eléments de photogrammétrie.

1892. *Meydenbauer.*—Das photographische Aufnehmen zu wissenschaftlichen Zwecken.

1893. *Pollack.*—Die Beziehungen der Photogrammetrie zu den topographischen Neuaufnahmen im bairisch-osterreichischen Grenzgebirge. Archiv fur die officiere des deutschen Reichsheeres.

1893. *Wang.*—Die Photogrammetrie oder Bildmesskunst im Dienste des Forsttechnikers.

1893-94. *Laussedat.*—Les applications de la perspective au lever des plans—Annales du Conservatoire des Art et Metiers.

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PHOTOGRAPHIC SURVEYING

CHAPTER I.

Descriptive Geometry.

1. Definition, planes of projection.—The object of descriptive geometry is to represent bodies and to solve problems on figures in space by means of their projections on certain planes called "*planes of projection.*"

2. Ground line.—For this purpose, two planes intersecting each other are employed; they divide space into four solid angles. Usually, one of the planes is vertical and the other one horizontal; their line of intersection is called "*ground line*" and is denoted by the letters XY.

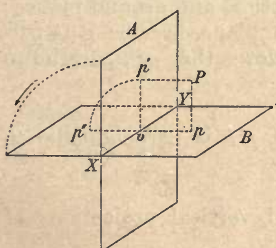

Fig. 1

3. Representation of a point. — Let XAY, Fig. 1, be the vertical plane, XBY the horizontal or ground plane, and P a point in space. From P, draw the perpendiculars Pp, Pp' to the ground and vertical planes; p is the *horizontal projection* of the point P and p' its *vertical projection.*

Let the vertical plane be revolved round the line XY as an axis, until it coincides with the ground plane; the point p' will fall at a point p'' such that the line pp'' will be perpendicular to XY.

For let a plane be drawn through Pp and Pp'; it is perpendicular to the ground plane as containing Pp and perpendicular to the vertical plane as containing Pp'; but when a plane is perpendicular to two other planes, it is perpendicular to their intersection, therefore, the plane pPp' is perpendicular to XY, and its traces op and op' on the ground and vertical planes, are also perpendicular to XY, since a line perpendicular to a plane is perpendicular to all the lines passing through its foot in the plane.

But op' being perpendicular to XY, op'' must also be perpendicular to XY; it follows that pop'' is a straight line perpendicular to XY. The ground plane is now as shown in Fig. 2, op' being the distance of

the point P from the ground plane and op its distance from the vertical plane; both points p and p' are on the same perpendicular to the ground line, as explained above.

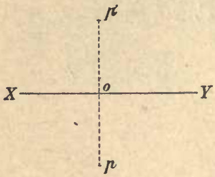

Fig. 2

It is usual to represent points in space by capital letters, the horizontal projections by italic letters and the vertical projections by the same italic letters accented.

It has been shown that the two projections of a point are on a perpendicular to the ground line. Inversely, any two points on a perpendicular to the ground line are the projections of a point of space.

For let the part of Fig. 2 above the ground line be revolved round the line XY until its plane be vertical as in Fig. 1. Through p draw a parallel to op' and through p' draw a parallel to op; they will meet in a point P.

But op' is perpendicular to XY by hypothesis and it is also perpendicular to op, since pop' is the angle of the vertical and ground planes; therefore op' is perpendicular to the ground plane, because it is perpendicular to two lines in that plane. It follows that pP, parallel to op' is perpendicular to the same plane.

In the same manner it may be shown that $p'P$ is perpendicular to the vertical plane:—

Therefore p and p' are the horizontal and vertical projections of the point P.

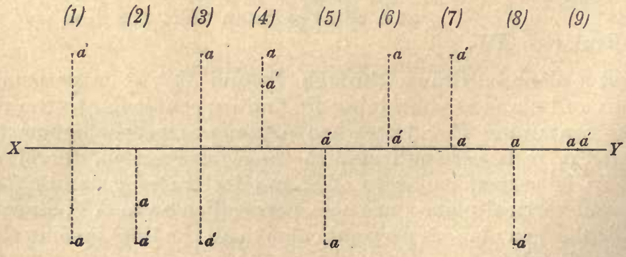

Fig. 3

Fig. 3 is the representation of a point in various positions.

(1.) Is a point in front of the vertical plane and above the ground plane.

(2.) Is in front of the vertical plane and below the ground plane.

(3.) Is behind the vertical plane and below the ground plane.

(4.) Is behind the vertical plane and above the ground plane.

(5.) Is in the ground plane in front of the ground line.

(6.) Is also in the ground plane but behind the ground line.

(7.) Is in the vertical plane above the ground line.

(8.) Is also in the vertical plane but below the ground line.

(9.) Is on the ground line.

4. Representation of a straight line.—If perpendiculars be drawn to a plane from every point of a straight line, the locus of the feet of the perpendiculars is a straight line and is the orthogonal projection of the first one.

The projection of a straight line may also be defined as the intersection of one of the planes of projection by a second plane perpendicular to the first one and containing the given line. This second plane is called the *projecting plane.*

A straight line is perfectly defined by its projections, because it is the intersection of the two projecting planes. There is, however, an exception when the given line is contained in a plane perpendicular to the ground line; the two projecting planes coincide and the projections of the line are not sufficient to define it; the traces must be given.

The "*traces*" of a line are the points where it intersects the planes of projection. The vertical trace c' (Fig. 4), being in the vertical plane, its horizontal projection is on the ground line, but it is also on the horizontal projection ab of the given line, therefore it must be at the intersection c of both lines. So by erecting at c a perpendicular to the ground line, the vertical trace will be found at c'.

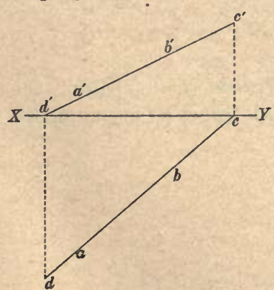

Fig. 4

Similarly, the horizontal trace is obtained by erecting at d' a perpendicular $d'd$ to the ground line; d is the horizontal trace.

Inversely, the projections of a line may be obtained from the traces. By drawing a perpendicular from the vertical trace c' to the ground line, a point c of the horizontal projection is obtained, which, joined to the horizontal trace d, gives the horizontal projection cd. The vertical projection is obtained in a similar manner by finding the vertical projection d' of the horizontal trace d and joining $c'd'$.

A straight line is defined by the projections of two of its points. Let aa', bb', Fig. 4, be the points. The projections of the line are $a'b'$, ab.

A straight line may occupy various positions with reference to the planes of projection; these positions are illustrated below.

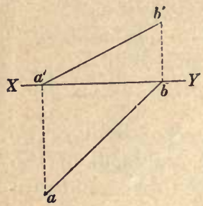

Fig. 5 shows a line intersecting the vertical plane at b', above the ground line and the ground plane at a, in front of the vertical plane.

Fig. 5

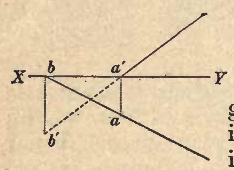

Fig. 6. The vertical trace, b', is below the ground line; the horizontal trace, a, is in front of it. The portion of the line between the traces is in the lower front dihedral angle.

Fig. 6

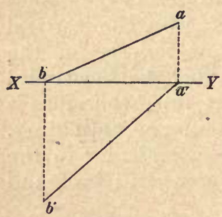

Fig. 7. The vertical trace, b', is below the ground line; the horizontal trace, a, is behind it.

Fig. 7

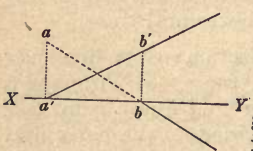

Fig. 8. The vertical trace, b', is above the ground line; the horizontal trace, a, is behind it.

Fig. 8

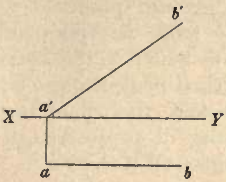

Fig. 9

Fig. 9. Line parallel to the vertical plane, with horizontal trace at a. In this case, the projecting plane through ab is parallel to the vertical plane; therefore its intersection with the ground plane, ab, is parallel to the ground line.

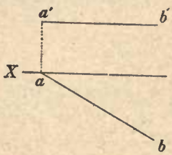

Fig. 10

Fig. 10. Horizontal line intersecting the vertical plane at a'. The projecting plane through $a'b'$ is parallel to the ground plane; therefore its intersection with the vertical plane, $a'b'$, is parallel to the ground line.

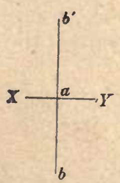

Fig. 11

Fig. 11. Any line in a plane perpendicular to the ground line. The horizontal and vertical projections coincide, and are on a perpendicular to the ground line. As explained above, the line in this case is not defined by its projections, which do not change, whatever may be the direction of the line in the projecting plane, but when the traces are given the line is defined.

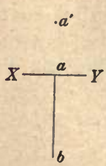

Fig. 12

Fig. 12. Line perpendicular to thevertical plane at a'. The vertical projection is a point, a', since any perpendicular to the vertical plane from a point of the given line will intersect the plane at a'. The horizontal projection, ab, is a line perpendicular to the ground line, because the projecting plane is perpendicular to the two planes of projection, and therefore is perpendicular to their intersection XY. There is no horizontal trace.

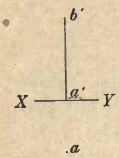

Fig. 13

Fig. 13. Line perpendicular to the ground plane at *a*. The perpendicular to the ground plane from any point of the given line intersects the plane at *a*, which is the horizontal projection of the line. The vertical projection, *a'b'*, is perpendicular to the ground line, because the projecting plane, being perpendicular to both planes of projection, is perpendicular to their intersection, the ground line.

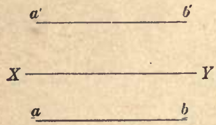

Fig. 14

Fig. 14. Line parallel to the ground line. In this case each of the projecting planes is parallel to the ground line; therefore their intersections with the corresponding planes of projection are also parallel to the ground line.

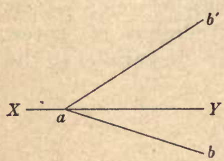

Fig. 15

Fig. 15. Line intersecting the ground line. The point of intersection, *a*, is at the same time the horizontal and the vertical trace of the line, and both projections intersect there.

When a line is in the ground plane its horizontal projection is the line itself and its vertical projection is the ground line.

When a line is in the vertical plane its vertical projection is the line itself and its horizontal projection is the ground line.

The ground line is its own horizontal projection and its own vertical projection.

5. Through a given point, to draw a parallel to a given line.— When two lines are parallel their projections of same denomination are also parallel, because their projecting planes, being perpendicular to the same plane of projection and passing through parallel lines, are themselves parallel to each other, and therefore their intersections with the plane of projection are parallel lines.

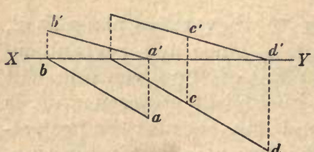

Fig. 16

It follows that when a parallel to a line *ab*, *a'b'* (Fig. 16) has to be drawn through a point *c*, *c'*, it is sufficient to draw through *c* a parallel to *ab* and through *c'* a parallel to *a'b'*; then *cd*, *c'd'* is parallel to *ab*, *a'b'*.

When two lines *ab*, *a'b'*; *cd*, *c'd'* (Fig. 17) intersect each other, the points of

intersection p and p' of the projections are on the same perpendicular to the ground line. It has been shown in § 3 that this is necessary in order that p and p' may represent a point in space.

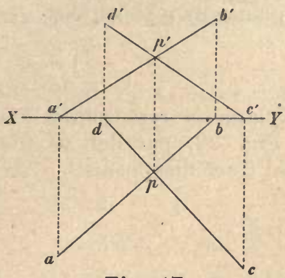

Fig. 17

It follows that when the points p and p' are not on the same perpendicular to the ground line, the lines ab, $a'b'$; cd, $c'd'$, do not intersect, that is to say they are not contained in one plane.

6. Representation of a plane.—A plane is represented by its traces on the planes of projection, that is to say by its intersections with the said planes. These traces meet in a point a, Fig. 18, of the ground line, which is the point where the plane cuts it. The vertical trace of the plane is aP', the horizontal trace is aP.

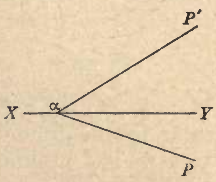

Fig.18

When the plane is vertical, its trace aP', Fig. 19, on the vertical plane, being the intersection of two vertical planes, is a vertical line.

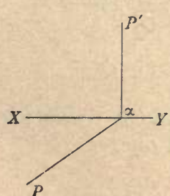

Fig. 19

But a vertical line is perpendicular to the ground plane and to all lines contained in this plane by which it is intersected; therefore it is perpendicular to the ground line.

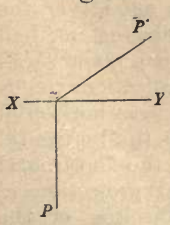

Fig.20

It may be shown in the same way that the horizontal trace aP, Fig. 20, of a plane perpendicular to the vertical plane, is a line perpendicular to the ground line.

Fig. 21

The plane may be parallel to the vertical plane, in which case the vertical trace disappears. The horizontal trace PQ, Fig. 21, is parallel to the ground line, because the two lines are the intersections of two parallel planes by a third one.

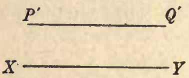

Fig. 22

Similarly, a horizontal plane is represented by its vertical trace $P'Q'$, Fig. 22, parallel to the ground line; the horizontal trace disappears.

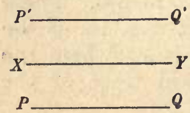

Fig. 23

When the plane is parallel to the ground line, the two traces PQ, $P'Q'$, Fig. 23, are also parallel to it.

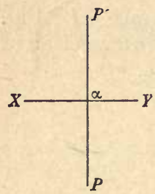

Fig. 24

A plane PaP', Fig. 24, perpendicular to the ground line, has its traces perpendicular to it. The ground line being perpendicular to the plane is perpendicular to all the lines passing through a in that plane and therefore is perpendicular to the traces aP, aP'.

Two parallel planes have their traces parallel, because the traces are then the intersections of two parallel planes by a third one.

7. Line contained in a plane.—A line contained in a plane has its traces on the traces of the plane, since any point of the planes of projection not on the traces is outside of the given plane. Hence the following method for finding a line contained in a plane PaP', Fig. 25, when one of its projections ab is given. The point a, where the horizontal projection of the line intersects the horizontal trace aP of the plane, is the horizontal trace of the line. Its vertical projection a' is a point of the vertical projection of the line. But the point of intersection b of ab

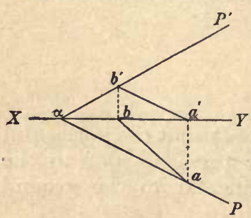

Fig. 25

with the ground line is the projection of a point of the line AB contained in the vertical plane, that is, the projection of the vertical trace of AB; then if at b a perpendicular bb' be erected to XY, its intersection b' with aP' will be the vertical trace of AB and the vertical projection will be obtained by joining $a'b'$.

8. Point in a plane.—When a point M is contained in a plane PaP', Fig. 26, one of the projections, m, of the point is sufficient to define it.

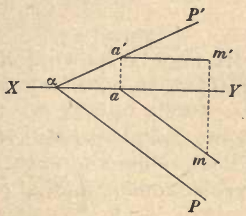

Fig. 26

To find the other projection, m', of M, let a horizontal line be drawn through M in the plane PaP'; its horizontal projection is a line ma parallel to aP, its vertical trace is at the intersection a' of the vertical trace of the plane PaP' with a perpendicular at a to the ground line and its vertical projection is a line $a'm'$ parallel to XY. The vertical projection of M is then found by drawing through m a perpendicular to XY and producing it to its intersection with $a'm'$. The point mm' being on the line am, $a'm'$, is in the plane PaP'.

It is not necessary that the line drawn through M be horizontal; any other line might be employed, but it is more convenient to use a line parallel either to the vertical or to the ground plane.

9. Through a point, to draw a plane parallel to another plane.—Let a point mm', Fig. 27, and a plane PaP' be given; through mm', it is required to draw a plane parallel to PaP'. Through mm', draw a line parallel to Pa; its horizontal projection is a parallel through m to Pa and its vertical projection a parallel through m' to the ground line (§ 5). Find the vertical trace a' by erecting at a a perpendicular to the ground line and producing it to its intersection with $m'a'$. Then through a' draw $\beta Q'$ parallel to aP' and through β draw βQ parallel to aP. The plane $Q\beta Q'$ is parallel to the plane PaP', and it contains the point mm', since it contains the line ma, $m'a'$.

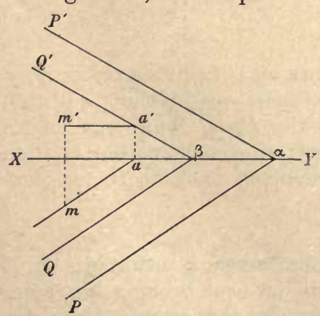

Fig. 27

10. Line perpendicular to a plane.—Let the line ma, $m'b'$, Fig. 28, be perpendicular to the plane PaP'; the projecting plane MA ma is perpendicular to the ground plane; it is also perpendicular to the plane PaP', since it contains a line MA perpendicular to this plane; therefore it is perpendicular to the intersection aP of these two planes and inversely the intersection aP is perpendicular to the projecting plane. But being perpendicular to the plane, aP is perpendicular to all lines passing through a and contained in the plane, therefore it is perpendicular to am.

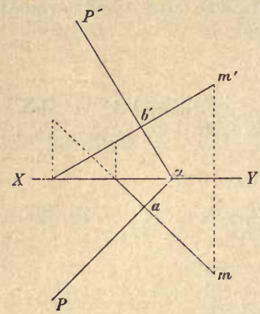

Fig. 28

In the same manner it may be shown that $b'm'$ is perpendicular to aP'.

So when through a point mm', it is required to draw a line perpendicular to a plane, perpendiculars ma and $m'b'$ to the traces of the plane are drawn through the projections m and m' of the point.

To draw through a given line a plane perpendicular to a given plane, a line perpendicular to the plane is drawn, as explained above, from any point of the given line, and then a plane is drawn through the two lines by joining the traces of same denomination of the lines.

When it is required to draw through a given point a plane perpendicular to a given line, perpendiculars to the projections of the line are drawn from any point of the ground line; they represent the traces of a plane perpendicular to the given line and there remains only to draw a plane parallel to the first one and passing through the given point as explained in § 9.

11. Revolving a plane upon one of the planes of projection.—For making constructions in a plane other than one of the projection planes, it is often convenient to revolve the plane round one of its traces upon the ground or vertical planes; the construction is then effected and, if necessary, the plane is revolved back to its original position.

The problem can always be reduced to finding the position of a point M of a plane PaP', Fig. 29, after this plane has been brought into coincidence with one of the planes of projection, the ground plane for instance, by a revolution round its horizontal trace aP.

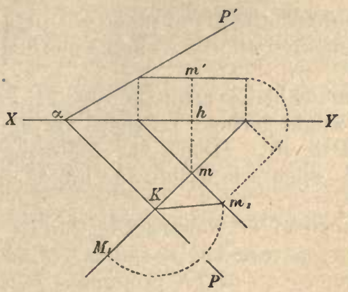

Fig. 29

From the point m, draw a perpendicular mK to the trace aP of the plane and join in space MK and Mm. The plane MKm is perpendicular to the ground plane as containing Mm; hence it contains the vertical line at K to which aK is perpendicular. But aK is also, by construction, perpendicular to Km, therefore it is perpendicular to the plane mKM and to KM which is in this plane. Consequently when the plane is revolved round its trace, M will fall on a perpendicular KM_1 to aP.

Let us suppose now that the triangle MKm be revolved round Km on the ground plane. The angle KmM being a right angle, the side mM will fall in mm_1, parallel to aP; mm_1, which is the height of M above the horizontal plane, is equal to hm'. The triangle Kmm_1, can therefore be constructed and by taking KM_1 equal to Km_1 the position of M_1 is obtained.

The construction lines on the figure are merely to show that mm_1 is made equal to hm', KM_1 equal to Km_1, and that the point mm' lies in the plane PaP' (§ 8).

A similar construction would be employed to revolve a plane upon the vertical plane.

It may be observed that the angle mKm_1 is the angle of the given plane with the ground plane.

The position of a line revolved upon the horizontal plane is determined by finding the positions of two of its points; its traces for instance. Let ab, $a'b'$, Fig. 30, be the line and PaP' the plane. From b draw a perpendicular bK to aP. The same demonstration as in the case of Fig. 29 shows that aP is perpendicular to the line Kb' in space, therefore b' in its revolution round aP will fall on Kb produced, and KB_1 will be equal to Kb'. But Kb' is the hypothenuse of the right angle triangle Kbb', which can be constructed at Kbb_1; by making KB_1 equal to Kb_1 the position of B_1 is obtained. The position of the horizontal trace a of the given line has not changed; therefore this line after its revolution will fall in aB_1.

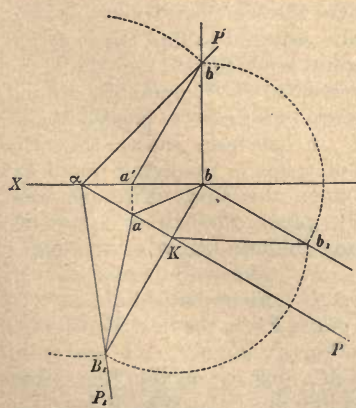

Fig. 30

Here again the angle bKb_1 is the angle of the given plane with the ground plane, and the construction indicated affords a simple method of finding it.

The line aP_1 is the position of the vertical trace after the revolution of the plane ; the angle PaP_1 is the angle formed in space by the traces of the plane, and aB_1 is equal to ab'. Hence the following construction to find the revolved line, when a is within the limits of the drawing :—

Draw bK perpendicular to aP and instead of constructing the triangle Kbb_1, describe a circle with a as a centre and ab' as radius. Join to a the point of intersection B_1 of the circle with bK produced ; B_1a is the revolved line, and aP_1 the revolved trace of the plane PaP'.

To revolve a plane back into its original position, inverse constructions are employed. Let it be required, for instance, to find the projections of the point M_1 (Fig. 29) when the plane PaP_1 is revolved back to PaP'. The angle of PaP' with the ground plane is first determined by the construction given above ; then from M_1 a perpendicular is drawn to aP, and at the point of intersection K an angle mKm_1 is constructed equal to the inclination of the given plane on the ground plane. Km_1 is taken equal to KM_1, and from m_1 a perpendicular m_1m is drawn to M_1m ; m is the horizontal projection of M, and mm_1 its height above the ground plane, from which the vertical projection is easily found.

A line is revolved back into its original position by repeating in inverse order the constructions given for revolving it upon the projection plane. Let aB_1 (Fig. 30) be the line ; from B_1 and a draw the perpendiculars B_1b and aa' to aP and XY respectively. At b erect a perpendicular bb' to XY, produce it to its intersection b' with aP', and join ab, $a'b'$, which are the projections required.

The constructions are simplified when the vertical trace has been revolved on the ground plane. Let it be required to find the position of the point mm' (Fig. 31) on the plane PaP' revolved in PaP_1 upon the ground plane. From m draw a perpendicular mn to aP ; it has been shown that the point M of space in revolving round aP will fall upon this line. Through mm' and in the plane PaP' draw a line ab, $a'b'$, cutting the two traces of the plane (§ 7) ; on aP_1 take aA_1, equal to aa', and join A_1b. As explained above, A_1b is, in the plane PaP_1 the line represented in projection at ab, $a'b'$, and the point required, M_1 must be on this line. But it has been shown that it is also on the line mn ; therefore it is at the intersection of both lines, in M_1.

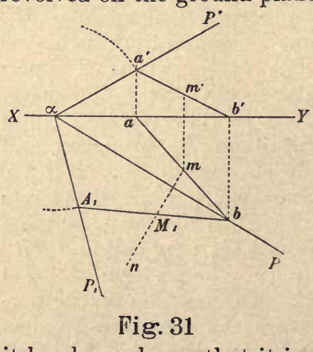

Fig. 31

To revolve back this point into its original position, a line A_1b, cutting the traces aP_1 and aP, is drawn through M_1; aa' is taken equal to aA_1, and perpendiculars $a'a$ and bb' are drawn from a' and b to the ground line; ab and $a'b'$ are the projections of the line bA_1 when revolved back to its original place. A perpendicular to aP is next drawn from M_1; its intersection with ab gives the horizontal projection m of the point M; the vertical projection is obtained by drawing through m a perpendicular to the ground line and producing to its intersection m' with $a'b'$.

Instead of the line ab, $a'b'$, a parallel to the vertical plane may be employed. Let mm', Fig. 32, be the point, PaP' the given plane and PaP_1 the same plane revolved upon the ground plane. From m draw a perpendicular mn to aP: the point M_1 will fall on this line. Then through mm', draw ab, $a'b'$, parallel to the vertical trace aP' of the plane (§ 5). When PaP' is revolved, this line still remains parallel to aP' and as its trace a does not move, the line falls in aB_1, parallel to aP_1. The point M_1 is on aB_1, but it is also on mn, therefore it is at the intersection M_1 of mn and aB_1.

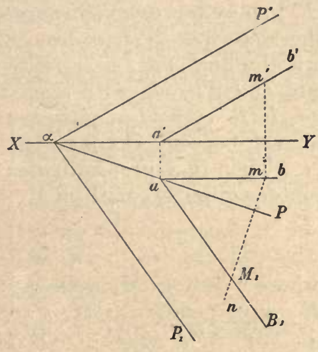

Fig. 32

To find the projections of the point M of space when it is given revolved on the ground plane in M_1, draw through M_1 a parallel aB_1 to the trace aP_1 and a perpendicular M_1m to aP. Through a draw ab parallel to the ground line; it is the horizontal projection of a line parallel to the vertical plane and passing through the point M. But the horizontal projection m of M is also on the line M_1m, therefore it is at the intersection of M_1m and ab.

The vertical projection m' of M is on the perpendicular mm' drawn through m to the ground line; it is also on the vertical projection $a'b'$ of the parallel to the vertical plane, which is obtained by drawing from a a perpendicular aa' to XY and through a' a parallel to the trace aP'. The intersection of $a'b'$ and mm' gives the vertical projection m' of M.

The constructions are still further simplified when the given plane is perpendicular to one of the planes of projection.

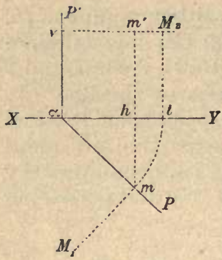

Fig. 33

Let PaP', Fig. 33, be a plane perpendicular to the ground plane and mm' a point of the plane. The point M in space is on the vertical line passing through m, which line is in the plane PaP' and is perpendicular to the horizontal line aP. Therefore, when PaP' is revolved round aP, the line mM remains perpendicular to aP and the point M falls in M_1 at a distance mM_1 from aP equal to the height of M above the ground plane. But this height is hm', therefore to determine the point M_1 draw at m a perpendicular to aP and take mM_1 equal to hm'.

Instead of revolving the plane round aP, it may be revolved round aP' on the vertical plane. The point M then describes in space an arc of circle of which the vertical projection is the line $m'M_2$ parallel to XY, and the horizontal projection is an arc of circle mt, with a as a centre and am as radius. When the plane PaP' coincides with the vertical plane, the point M of the plane must be somewhere on the line $m'M_2$ and its horizontal projection is t. Then if a perpendicular to XY be erected at t and produced to its intersection M_2 with $m'M_2$, M_2 will be the required point.

To find the projections of the point M, whose position M_1 revolved on the ground plane is given, draw from M_1 a perpendicular M_1m to aP and from m a perpendicular mm' to XY. Take hm' equal to mM_1, height of the point M above the ground plane ; m, m' are the projections of the point.

The projections of M_2 are found by drawing through M_2 a parallel M_2v to XY, which is the vertical projection of the arc of circle described by M_2 when revolved back to its original position ; take am equal to the distance M_2v of M_2 from the trace aP' and through m draw the perpendicular mm' to XY. m, m' are the projections of the point.

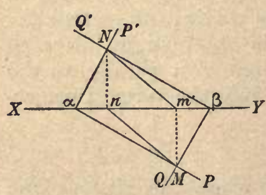

Fig. 34

12. Intersection of two planes. Let PaP' and $Q\beta Q'$, Fig. 34, be two planes : the points M and N where the traces of the planes meet, are the traces of the line of intersection of the planes. The projections Mn, $m'N$ of the intersection are found as explained in § 4 by letting fall the perpendiculars Mm' and Nn to the ground line and joining Mn, $m'N$.

13. The intersecting planes are both parallel to the ground line.—Let PQ, $P'Q'$, RS, $R'S'$, (Fig. 35), be the traces of two intersecting planes parallel to the ground line; the construction given in § 12 does not apply and recourse must be had to an auxiliary plane. Draw a plane TOT' perpendicular to the ground line. The line of intersection of the two given planes is parallel to the ground line and so are its projections. If the projections m and m' of the point M where this line intersects the plane TOT' were known, the projections of the line itself would be obtained at once by drawing through m and m' parallels to the ground line.

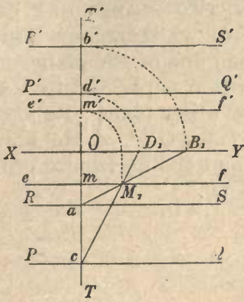

Fig. 35

To obtain M, let us revolve TOT' around OT upon the ground plane; the intersection of TOT' and $PQ\ P'Q'$, of which the traces are c and d', will fall in cD_1; OD_1 being equal to Od'. Similarly the intersection of TOT' and $RS\ R'S'$ will fall in aB_1, OB_1 being equal to Ob' and the point M will come in M_1, at the intersection of cD_1 and aB_1. From M_1 draw a perpendicular to OT; the point of intersection m is the horizontal projection of M. The vertical projection is obtained by making Om' equal to mM_1, this being the height of M above the ground plane. Then through m and m' draw the parallels ef, $e'f'$, to the ground line; they are the projections of the line of intersection.

14. The intersecting planes cut the ground line at the same point.—Let PaP', QaQ', (Fig. 36), be two intersecting planes cutting the ground line at a. Draw a plane TOT' perpendicular to this line; a is a point of the line of intersection of the planes and if the projections m and m' of the point M where this line cuts the plane TOT' were known, the projections of the intersection would be obtained by joining am and am'.

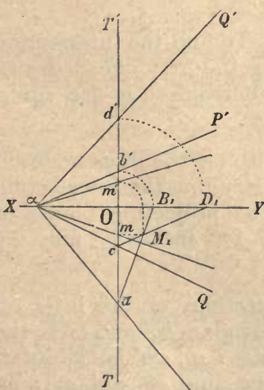

Fig. 36

Let us revolve the plane TOT' around OT: the intersection of TOT' and PaP', of which a and b' are the traces, will fall in aB_1, OB_1 being equal to Ob'. Similarly the intersection of TOT' and QaQ' will fall in cD_1, OD_1 being equal to Od', and M will come in M_1, at the intersection of aB_1 and cD_1. From M_1, draw a perpendicular M_1m to OT, the point m is the horizontal projection of M. The vertical projection m' is obtained by making Om' equal to mM_1. Then draw am and am' which are the projections of the intersection.

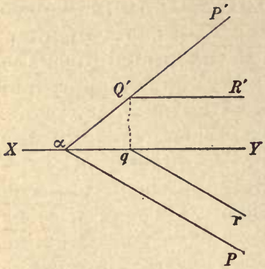

Fig. 37

15. Intersection of two planes, one of which is horizontal or parallel to the vertical plane.—When one of the planes is horizontal, the intersection is parallel to the horizontal trace of the other plane: its vertical projection is the trace $Q'R'$, of the horizontal plane (Fig. 37) and the horizontal projection a parallel qr to aP.

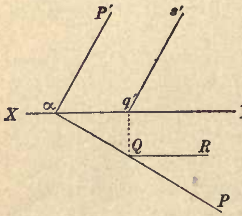

Fig. 38

In the case of a plane parallel to the vertical plane (Fig. 38), the horizontal projection of the intersection is the trace QR of the vertical plane. The vertical projection is a parallel $q's'$ to the vertical trace aP' of the other plane.

16. Planes perpendicular to one of the planes of projection.—When the two planes are both perpendicular to one of the planes of projection, their intersection is also perpendicular to this plane and its projection on it is the point where the traces of the planes meet. The projection on the other plane is a perpendicular to the ground line passing through the above point.

17. Intersection of a line and a plane.—To find the intersection of a line and a plane, another line intersecting the first one is drawn in the plane; the point required is the intersection of the two lines.

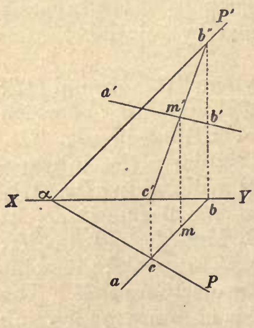

Fig. 39

Let ab, $a'b'$, (Fig. 39), be the line and PaP' the plane. For auxiliary line the intersection of PaP' by one of the projecting planes of the given line, abb', for instance, may be employed.

To obtain the projections of this intersection, draw the perpendiculars bb'' and cc' to the ground line and join $c'b''$: cb, $c'b''$ is the intersection. It meets the line ab $a'b'$ at mm' which is the point where the line cuts the plane PaP'.

R

18. Intersection of three planes.—The intersection of three planes may be found either by constructing the line of intersection of two of the planes, and then determining the point where this line cuts the third plane, or by constructing the lines of intersection of one of the planes with each of the others; the point where the two lines meet is the point of intersection of the three planes.

19. Through a point, to draw a straight line which will meet two given lines.—To draw through a point a straight line which will meet two given lines not in the same plane, a plane is passed through the point and one of the lines. The point where the second line pierces the plane is ascertained (§ 17), and by joining this point of intersection to the given point, the required line is obtained.

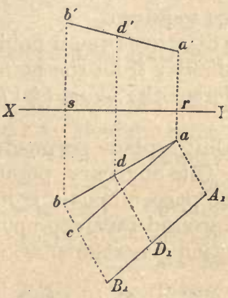

Fig. 40

20. Distance of two points.—Let aa', bb', (Fig. 40), be two points; to obtain their distance, one of the projecting planes of the line AB may be revolved about its trace upon the corresponding projection plane.

Let us revolve, for instance AB ab around ab. The point A will fall in A_1 on a perpendicular aA_1 to ab, the line aA_1 being the height of A above the ground plane, that is the distance ra'. Similarly B will fall in B_1, on a perpendicular bB_1 to ab, and at a distance from b equal to sb'. The required distance of the points is A_1B_1.

The construction may be somewhat simplified by observing that if a line be drawn through a parallel to A_1B_1, its length ac is equal to A_1B_1; therefore, instead of constructing the trapezoid aA_1B_1b, it is sufficient to erect a perpendicular to ab at b and to lay off on it a distance bc equal to the difference between sb' and ra'.

21. To lay off a given length on a line.—The construction given in § 20 may be employed for laying off a given length on a line AB (Fig. 40). Turn the projecting plane on the line ab as an axis and lay off the required length A_1D_1 on A_1B_1. Then revolve the projecting plane back to its natural position; the horizontal projection of D will be at d, foot of the perpendicular drawn from D_1 to ab, and its vertical projection will be at d', intersection of $a'b'$ by a perpendicular through d to the ground line.

22. Distance from a point to a line.— The distance from a point to a straight line is obtained by passing a plane through the line and the point, and revolving it upon one of the planes of projection. Let ab, $a'b'$, be the line and mm' the point (Fig. 41). Through mm' draw a parallel cd, $c'd'$ to ab, $a'b'$; the line ac is the horizontal trace of the plane containing the two parallel lines. Revolve this plane around its trace ac, until it coincides with the ground plane (§ 11).

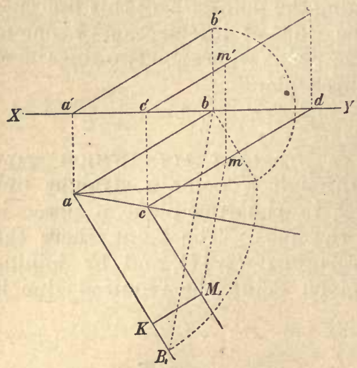

Fig. 41

Let aB_1 and M_1 be the revolved positions of ab' and M. From M_1 let fall a perpendicular M_1K to aB_1; it is the distance required.

23. Distance from a point to a plane.—The distance from a point to a plane may be obtained by dropping a perpendicular from the point to the plane (§ 10), finding the point where it pierces the plane (§ 17) and determining the distance of the two points.

It is more convenient to pass through the point a plane perpendicular to one of the traces of the given plane. This auxiliary plane, being perpendicular to the other one, contains the perpendicular from the point to the given plane; after revolving it around its trace upon one of the planes of projection, a simple construction gives the solution of the problem.

Let PaP' (Fig. 42) be the plane and mm' the point. Through mm' pass the plane $Q\beta Q'$ perpendicular to aP' and revolve it around βQ upon the ground plane. The point A describes the arc of circle AA_1, and BA_1 is the intersection of the two planes revolved upon the ground plane.

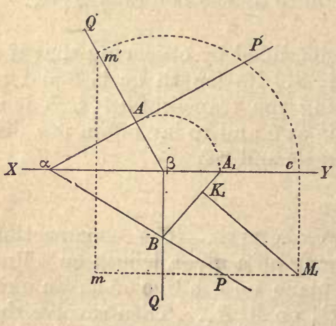

Fig. 42

The point M is on a parallel to βQ passing through m'. In revolving the auxiliary plane m' describes the arc of circle $m'c$ and the line $m'M$ falls in cM_1, still parallel to βQ. The point M remaining during the revolution of the plane at a constant distance from the vertical plane falls on a parallel to the ground line passing through m; therefore M_1 comes at the intersection of cM_1 and mM_1. There remains only to let fall a perpendicular M_1K_1 from M_1 to BA_1; it is the distance required.

24. Distance of two parallel planes.—The distance of two parallel planes may be obtained by intersecting them by a third plane perpendicular to both and revolving it upon one of the planes of projection.

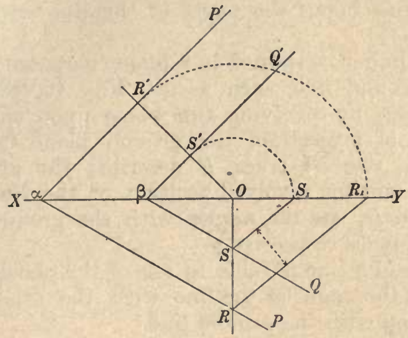

Fig. 43

Let $Pa P'$, $Q\beta Q'$ (Fig. 43) be the parallel planes. Draw a plane ROR' perpendicular to the vertical traces and revolve it upon the ground plane around OR as an axis. The points R' and S' describe the arcs of circles $R'R_1$, $S'S_1$; the lines RR_1 and SS_1 being the intersections of the given planes by the auxiliary one. These lines are parallel and their distance is the distance of the planes.

25. Distance of two straight lines.—Let AB and CD (Fig. 44) be two straight lines not contained in one plane; it is required to find their shortest distance. This distance is the length of the perpendicular to both lines. Through any point of AB, A for instance, draw a parallel AF to CD, and from a point G of CD let fall a perpendicular GH on the plane BAF. Through the foot of GH in the plane BAF draw a parallel HK to AF and through K another parallel KM to HG. The line KM is perpendicular to both lines.

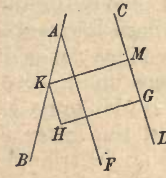

Fig. 44

Although presenting no difficulty, the construction requires many lines and is omitted here.

26. Angles of a line with the planes of projection.—Let it be required to find the angles formed by the line ab, $a'b'$, (Fig. 45), with the planes of projection.

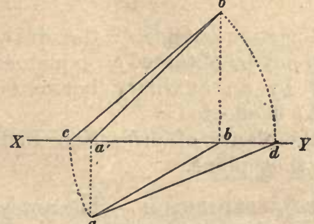

Fig. 45

The angle of the line with the ground plane is the same as with the line ab, since the plane $b'ab$ is perpendicular to the ground plane. This angle can be obtained by revolving the triangle $b'ba$ around $b'b$ as an axis upon the vertical plane. The vertex a describes the arc of circle ac and the triangle comes in $b'bc$, the angle at c being the angle of the line with the ground plane.

Similarly the angle with the vertical plane is obtained by revolving the triangle $aa'b'$ upon the ground plane around aa' as an axis. The vertex b' comes in d, the angle ada' being the angle of the line with the vertical plane.

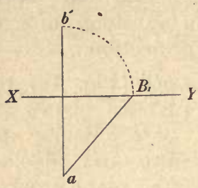

Fig. 46

When the line is contained in a plane perpendicular to the ground line, such as ab', Fig. 46, the angles are found by revolving this plane upon one of the planes of projection, the ground plane for instance: the vertical trace, b' describes the arc of circle $b'B_1$ and the revolved position of the line is aB_1; a and B_1 are the angles with the ground and vertical planes respectively.

In the case of a line parallel to one of the planes of projection, the angle of the line with the other plane is the angle of its projection with the ground line.

27. Angle of two lines.—To find the angle formed by two intersecting lines, their plane is revolved about its trace upon one of the planes of projection. Let ab, $a'b'$; cd, $c'd'$, (Fig. 47), be the lines. The horizontal trace of their plane is the line ac passing through their traces; it forms with the two lines a triangle aMc in which M is the angle to be found.

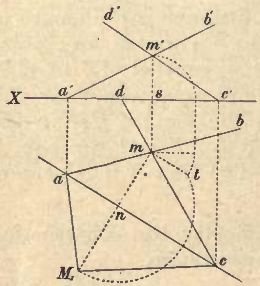

Fig. 47

Revolve this triangle around ac upon the ground plane; the point M moves in the plane perpendicular to ac whose horizontal trace is the perpendicular mn to ac; it will therefore fall in M_1, somewhere on mn produced. The distance nM_1 is the same as the distance from n to M and the latter is the hypothenuse of the right angle triangle Mmn. The side Mm of this triangle being the height of M above the ground line, the triangle can be constructed by erecting at m a perpendicular to mn and laying off mt equal to $m's$; M_1 is then determined by making nM_1 equal to nt. Joining M_1a and M_1c, the angle required is aM_1c.

It may happen that the traces of the lines are outside of the drawing, and that the trace of their plane can not be obtained as explained above. In that case, the lines are cut by an auxiliary horizontal plane on which the construction of Fig. 47 is effected.

When the lines are parallel to one of the planes of projection, their angle is the angle of their projections on that plane.

28. Angles of a plane with the planes of projection.—The angles of a plane with the planes of projection are obtained by cutting it by auxiliary planes perpendicular to the traces. Let PaP', (Fig. 48), be

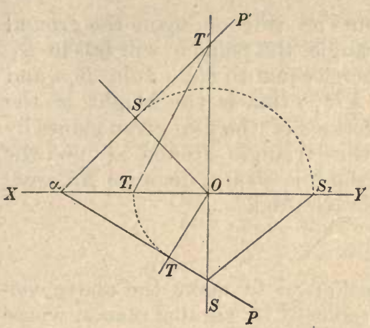

Fig. 48

the plane. Draw a plane TOT' perpendicular to aP: its intersections with the planes of projection and the given plane form a right angled triangle TOT' in which the angle at T is the angle of PaP' with the ground plane. Revolve this triangle upon the vertical plane around OT' as an axis: T describes an arc of circle TT_1, of which O is the centre, and the triangle comes in $T'OT_1$, the angle T_1 being the angle of PaP' with the ground plane.

Similarly, the angle with the vertical plane is obtained by drawing the plane SOS' perpendicular to aP' and revolving the triangle SOS' upon the ground plane in SOS_1. The angle at S_1 is the angle of PaP' with the vertical plane.

The line $T'T$ is the line of greatest declivity of the plane PaP': any other line contained in the plane PaP' and not parallel to $T'T$ forms with the ground plane an angle smaller than $T'TO$.

29. Angle of two planes.—Let PaP', $Q\beta Q'$, (Fig. 49), be two planes, of which it is required to find the angle. Their intersection is projected horizontally in ab. Cut the planes by another one perpendicular to both; it is perpendicular to their intersection and consequently the horizontal trace cd is perpendicular to ab. The intersections of this plane with the two given planes form with the trace cd a triangle in which the angle opposite cd is the angle of the two planes.

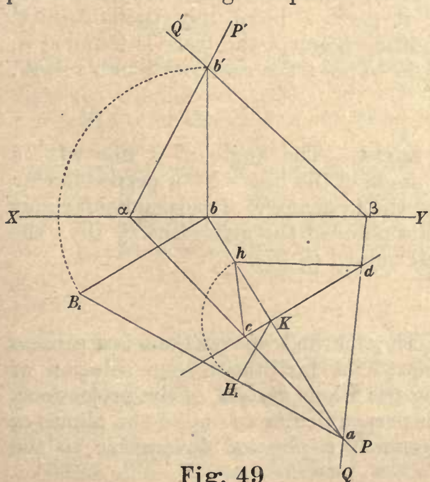

Fig. 49

The intersection of the auxiliary plane with the projecting plane abb' is the perpendicular let fall from the vertex of the triangle on cd because cd being perpendicular to the projecting plane is perpendicular to all lines contained in it passing through its foot K. The same intersection is also perpendicular to the intersection ab' of the two given planes, because ab' being perpendicular to the auxiliary plane, is perpendicular to all lines contained in that plane by which it is intersected.

Now revolve the triangle abb' about its side ab upon the ground plane. The angle at b being a right angle, the point b' will fall in B_1 on a perpendicular to ab at b, bB_1 being equal to bb'. Join B_1a and let fall on it from K a perpendicular KH_1 : this is the height of the triangle formed by cd and the intersections of the two given planes by the auxiliary plane. Then revolve this triangle around cd upon the ground plane; its vertex will fall on the line ab at a distance Kh equal to KH_1. Join hc, hd; chd is the angle required.

When the planes are in such a position as to make the above construction inconvenient, they may be replaced by parallel planes, whose positions are selected at pleasure. This may be done, for instance, when the planes cut the ground line at the same point or when their traces do not meet within the limits of the drawing.

When the planes are both parallel to the ground line, the construction is the same as in Fig. 35 ; aM_1c is the angle of the planes.

30. Through a given line in a plane to draw another plane making a certain angle with the given plane.—The converse problem consists in drawing through a given line of a plane, another plane making with the first one a given angle. The construction is the same as in Fig. 49, but is inverted. The given line is the intersection of the two planes ; the triangle chd is constructed by means of the line KH_1 and the angle h; it gives a point d of the horizontal trace of the plane required. Another point of the trace is found at a, then join ad, produce to β and join to $\beta b'$: the required plane is $a\beta b'$.

31. Angle of a line with a plane.—The angle of a line with a plane is the complement of the angle of the line with a perpendicular to the plane. So in order to find the first angle, a perpendicular may be erected to the plane through a point of the given line (§ 10) ; the angle of the two lines is then determined (§ 27).

32. Method of rotations.—The method of rotations is a process employed in Descriptive Geometry for facilitating the solution of problems. It consists in rotating the whole system of the projections, or only part of it, around an axis perpendicular to one of the planes of projection, until the system assumes a position favourable to the solution of the problem.

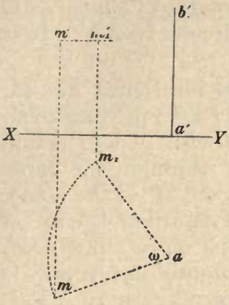

Fig. 50

33. Rotation of a point—Let it be required to rotate a point mm', (Fig. 50), through an angle ω, around a vertical axis a, $a'b'$. The projection m, describes an arc of circle mm_1, with centre at a and subtending an angle equal to ω. But the point M, during its motion, remains at the same distance from the ground plane; therefore its vertical projection m', travels on a parallel $m'm'_1$ to the ground line. So when the point m has described the arc ω, the point m' is in m'_1, at the intersection of the perpendicular to the ground line through m_1 with the parallel to the same line through m'.

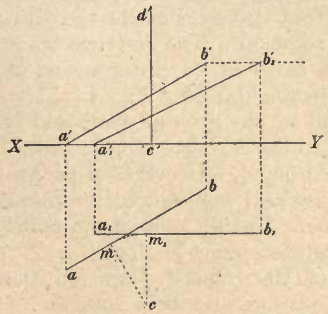

Fig. 51

34. Rotation of a line.—Let ab, $a'b'$, (Fig. 51), be a straight line to be rotated around a vertical axis c, $c'd'$ until parallel to the vertical plane. From c let fall the perpendicular cm on ab, and rotate the projecting plane containing ab around the axis. The point m describes an arc of circle and stops at m_1 on the perpendicular to the ground line drawn through c. The projecting plane is then parallel to the vertical plane and so are the lines ab and aB. The new position of ab is obtained by drawing through m_1 a parallel a_1b_1 to the ground line and making $a_1m_1 = am$, $b_1m_1 = bm$. The height above the ground plane of the points A and B of the given line does not change during the revolution around the axis; the vertical projection a' of the trace a therefore moves on the ground line and the projection b' on a parallel $b'b'_1$ to the ground line. But a'_1, the new vertical projection of a, must be on the perpendicular through a_1 to the ground line and since it is also on the ground line, it must be at their intersection in a'_1. Similarly, b'_1 must be on the perpendicular $b_1b'_1$ to the ground line and also on the parallel $b'b'_1$, therefore, it must be at their intersection b'_1. The rotated vertical projection is then $a'_1b'_1$.

35. Rotation of a plane—A plane may be rotated by turning three of its points, not on a straight line (§33), or a point and straight line, both in the plane, or two of its lines (§34). The following method is a simple one :—

PaP', (Fig. 52), is a plane to be rotated until perpendicular to the vertical plane, about a vertical axis of which the horizontal trace is c.

From c let fall a perpendicular cd on aP and rotate aP until cd is parallel to the ground line: a_1P_1 is then perpendicular to XY. It is the rotated horizontal trace of the plane.

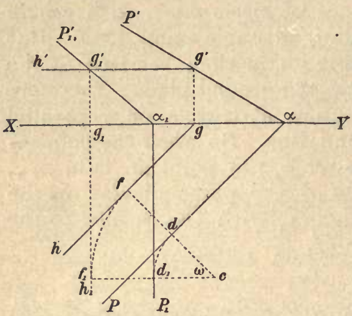

Fig. 52

Now draw any horizontal line gh, $g'h'$, in the plane PaP'; produce cd to its intersection f with gh and rotate the line gh, $g'h'$ through the same angle, ω, as the trace aP of the plane. The point f of cd describes the arc of circle ff_1 and stops on cd_1 produced. The rotated horizontal projection is the line g_1h_1 perpendicular to XY.

To obtain the vertical projection, it must be observed that the height of gh, $g'h'$, above the ground plane is gg' and that it does not change during the rotation. The vertical trace g' moves on the parallel $g'h'$ to XY, and stops at the intersection of $g'h'$ and g_1h_1 produced, since g_1h_1 is perpendicular to XY.

The rotated line $g_1h_1g'_1$, is still parallel to the ground plane and is now contained in a plane perpendicular to the vertical plane; therefore it is itself perpendicular to the vertical plane. Its vertical projection is the point g'_1, which is also its trace and consequently a point of the vertical trace of the rotated plane. But a_1 is another point of the new vertical trace, therefore the rotated plane is $P_1a_1P'_1$.

The angle $g_1a_1g'_1$ is the inclination of the rotated plane on the ground plane: this inclination is the same before and after rotation.

The plane might now be brought parallel to the ground plane by a second rotation about an axis perpendicular to the vertical plane.

36. Distance of two points.—As an application of this method, the determination of the distance of two points may be given.

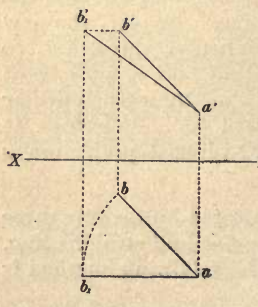

Fig. 53

Let aa', bb', (Fig 53), be the points. Rotate the vertical projecting plane containing a and b around the vertical line through a until it is parallel to the vertical plane. The point b describes an arc of circle bb_1 and stops at b_1 on the parallel ab_1 to XY; b' moves on a parallel to XY and stops at b'_1 at the intersection of the parallel $b'b'_1$ and the perpendicular $b_1b'_1$ to the ground line. The rotated line ab_1, $a'b'_1$ is parallel to the vertical plane; it is therefore equal to its vertical projection $a'b'_1$. The inclination of the line on the ground plane is equal to the angle formed by the vertical projection $a'b'_1$ and the ground line.

Another solution of this problem is given in § 20.

37. Solution of spherical triangles.—A spherical triangle may be assimilated to a trihedral angle by supposing the vertex of the angle to be at the centre of the sphere. The sides of the spherical triangle are then subtended by the plane angles of the faces of the trihedral angle and the angles of the triangle are the same as the dihedral angles of the trihedral angle.

As usual, the sides of the spherical triangle are designated by a, b and c, the opposite angles being A, B and C.

38. Given three sides, to find the angles.—The three sides of the triangle correspond to the three faces of the trihedral angle. Develop them on the ground plane, placing one of the edges, OQ, (Fig. 54), perpendicular to the ground line and revolve the faces a and c about the edges OQ and OR, upon the ground plane. The intersection of the trihedral angle by the vertical plane is the base of a pyramid of which O is the vertex and OQR one of the faces in its natural position. Since OQ is perpendicular to the vertical plane, the planes of the two faces intersecting along OQ are also perpendicular to the vertical plane, therefore OQP_1 is one of the faces of the pyramid, revolved upon the ground plane about OQ, and OP_1 is the third edge of the pyramid, the vertical trace of which is on the arc of circle described from Q as a centre with QP_1 as radius.

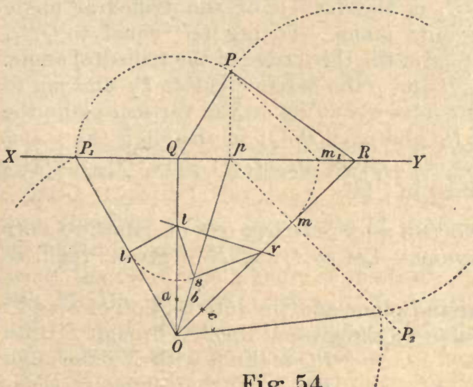

Fig. 54

The third edge of the pyramid is also shown in OP_2 which must be taken equal to OP_1; P_2, like P_1, is the vertical trace of the third edge OP revolved upon the ground plane. Let now the face c be revolved back to its natural position, by turning it about OR: the horizontal projection of P_2 will move on the perpendicular P_2m let fall from P_2 on OR, and when P_2 comes to its original place in the vertical plane, its horizontal projection will have moved along P_2m up to its intersection p with the ground line. The vertical trace P will therefore be on the perpendicular pP to the ground line, but being also on the arc of circle P_1 P, it is at their intersection.

Having now obtained the trace P of the edge OP on the vertical plane, the dihedral angle C is found at once in PQR, since both faces are perpendicular to the vertical plane.

Generally, only one angle is required: in making the construction, the edge corresponding to this angle is placed perpendicular to the ground line.

Should the other angles be wanted, A could be obtained from the triangle pmP revolved around Pp on the vertical plane; Pm_1p is the angle A of the spherical triangle. B is constructed as explained in § 29 or by any other method.

39. Given two sides and the included angle, to find the remaining side and angles.—Let a, b and C, be given; required c, A and B.

Place the intersection of a and b in OQ, (Fig. 54), perpendicular to XY, and the face b on the ground plane, draw QP making the angle PQY equal to C: QP is the vertical trace of the face a. Make the angle QOP_1 equal to a: QOP_1 is the face a of the trihedral angle revolved about OQ on the ground plane. Taking QP equal to QP_1, the point P is the vertical trace of the third edge of the trihedral angle.

To obtain A, let fall from P and p the perpendiculars Pp and pm to XY and OR respectively. Revolve about Pp on the vertical plane the triangle formed in space by Pp and pm: Pm_1p is the angle A.

Then produce pm and take mP_2 equal to m_1P: join OP_2: ROP_2 is c. B is obtained as explained in § 29.

40. Given two angles and the side opposite one of them, to find the remaining sides and angle.—Let a, A, and B be given: required C, b and c.

Place the face c on the ground plane and the intersection of a and c in OP, (Fig. 55), perpendicular to the ground line. Through P draw PQ making with XY the angle B; PQ is the vertical trace of the face a, since a and c are both perpendicular to the vertical plane. Draw OQ_1 making the angle a with PO; POQ_1 is the face a of the trihedral angle revolved upon the ground plane about OP as an axis, Q_1 is the revolved vertical trace of the edge OQ. Making then PQ equal to PQ_1 gives the trace Q. Through Q draw Qm_1 making the angle A

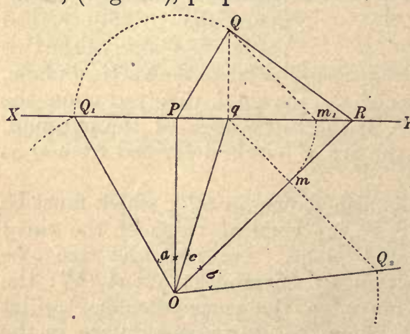

Fig. 55

with XY, and let fall the perpendicular Qq to XY. From q as a centre and qm_1 as radius, describe the arc of circle mm_1 and through O draw the tangent OR to the circle. The angle of the plane ORQ with the ground plane is equal to A, therefore ROQ is the face b of the trihedral angle and POR is the face c.

b and the angle C are obtained as in former cases.

41. Given two sides and the angle opposite one of them, to find the remaining side and angles.—Let a, b and B be given: required A, C and c.

Place the face a on the ground plane with the intersection of a and b in OP, (Fig. 56), perpendicular to XY. Make POR and POQ_1 equal to a and b respectively: POQ_1 is the face b of the trihedral angle revolved about OP on the ground plane, therefore the vertical trace Q of the edge opposite to a is on a circle described from P as a centre with PQ_1 as radius. Through P pass a plane perpendicular to OR: its horizontal trace is a line Pm perpendicular to OR and its vertical trace the perpendicular PS to XY. The intersections of this plane with the two planes of projection and the plane of the face c, form in space a triangle SPm in which P is a right angle and m is the angle B. Revolving this triangle about SP upon the vertical plane, in SPm_1, the point S is obtained. But S is a point of the vertical plane of projection and is also a point of the plane of the face c, therefore it is a point of the trace of the last plane. Joining then RS, the intersection of this line with the circle Q_1Q is the vertical trace of the edge of the trihedral angle opposite to a.

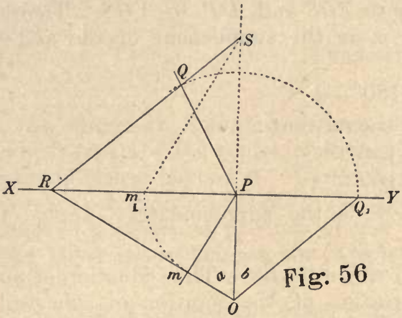

Fig. 56

A, C and c are now constructed as in former cases.

42. Other cases—supplementary triangles.—The other cases of spherical triangles are generally solved by the use of the supplementary triangles or trihedral angles. The direct solution, although possible, is not so convenient. The angles A_1, B_1, C_1, of the supplementary triangle are the supplements of the sides a, b, c, of the other triangle and the sides a_1, b_1, c_1, of the supplementary triangle are the supplements of the angles A, B, C, of the other one.

From any point O, (Fig. 57), in the interior of the trihedral angle, let fall perpendiculars OS, OT, OV, on the faces. The angle of OT and OV is the supplement of the angle of the planes to which they are perpendicular. But the angle of the planes is the angle B of the trihedral angle; therefore TOV or b_1 is equal to $180°—B$.

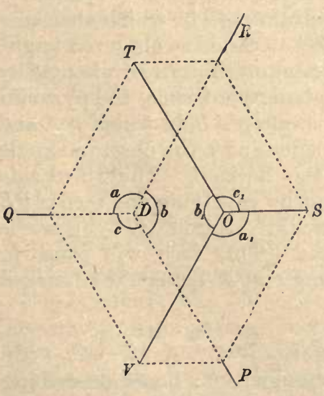

Fig. 57

Similarly: $TOS = c_1 = 180° - C$
$SOV = a_1 = 180° - A.$

The plane TOV containing perpendiculars to a and c, is perpendicular to both; therefore, it is perpendicular to their intersection DQ and conversely DQ is perpendicular to TOV. For the same reasons DR is perpendicular to TOS and DP to VOS. Therefore the angle of DQ and DR or a, is the supplement of the angle formed by VOT and TOS or A_1;

$$A_1 = 180° - a.$$

In the same manner, it may be shown that:

$$B_1 = 180° - b,$$

and $C_1 = 180° - c,$

Hence the trihedral angle $OTSV$ is the supplementary angle of $DPQR$.

43. Reduction of an angle to the horizon.—The reduction of an angle to the horizon is an application of the solution of spherical triangles. When an angle is observed between two points which are not in the horizontal plane of the observer, the observed angle requires a correction to reduce it to the angle formed by the projections of the points on the ground plane. For that purpose the observer measures the angular elevations or depressions of the points.

Take as vertical plane of projection the plane passing through the observer and one of the points. Assume any point P, (Fig. 58), as the place of observation and draw through it the lines PA and PB_1 making with the ground line angles equal to the elevations or depressions α and β of the points.

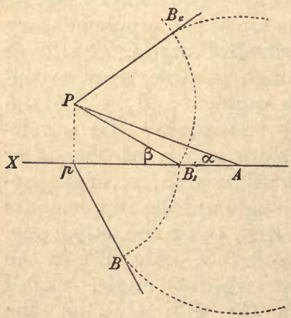

Fig. 58

The lines PA, PB and the vertical Pp form a trihedral angle in which the faces are $90° - \alpha$, $90° - \beta$ and the observed angle. A pyramid is cut off this trihedral angle by the ground plane, the base of the pyramid being the triangle $p\,AB$, in which $p\,A$ and $p\,B$ are two sides and p the observed angle reduced to the horizon. The third side may be found by revolving the face APB of the pyramid around AP, upon the vertical plane. This face will come in APB_2, the angle at P being the observed angle and

$$PB_2 = PB = PB_1$$

We have now the third side of the triangle AB: hence describing arcs of circles from A and p as centres with AB_2 and pB_1 as radii respectively, their intersection is the point B and ApB is the required angle.

CHAPTER II.

Perspective.

44. General Remarks.—Perspective is that branch of Geometry which treats of the representation by figures drawn on a surface of objects placed beyond it. Generally this surface is a vertical plane; it is called "*picture plane*." The figures drawn on it, according to the rules of perspective, produce on the eye, as far as form is concerned, the same impression as the objects themselves seen in their actual places.

Suppose a transparent plane surface, such as glass, placed between the eye and the objects to be represented. If the outlines of the objects seen through the glass could be traced on it, the image thus formed would be an exact perspective.

Consider the visual ray from the eye to a point of space: this ray pierces the picture plane in a second point, which is called the "*perspective*" of the first one.

The visual rays from the eye to all the points of a straight line form a plane whose intersection with the picture plane is the perspective of the line. Consequently, the perspective of a straight line is another straight line.

When the line is a curve, the visual rays to its various points form a conic surface whose vertex is at the eye and whose intersection with the picture plane is the perspective of the curve. A surface of the same nature is formed by the visual rays tangent to the visible outline of an object; the perspective of the object is the intersection of this surface by the picture plane.

45. Definitions.—The "*ground plan*" is the horizontal projection of the objects to be represented; thus for the perspective of a landscape, the ground plan is the topographical plan of the ground; for a building, it is the horizontal or ground plan of the building ($ABCD$, Fig. 59).

The "*ground plane*" is the plane on which the ground plan is placed ($KXsY$, Fig. 59). For a landscape, it may be, for instance, the horizontal plane passing through the datum point of the topographical plan and for a building, the basement or first floor plane. Any horizontal plane may, however, be used as ground plane, provided its altitude be taken into account; the ground plan does not change, whatever the altitude may be.

The "*elevation*" is the vertical projection of an object; the elevations of a building are those plans of the building which show the front, rear, or sides.

The "*picture plane*," as already explained, is the plane on which the perspective is drawn (*FFXY*, Fig. 59). Generally, it is vertical and placed between the eye and the object to be represented, but none of these rules is absolute. Perspectives are sometimes drawn on planes which are not vertical and objects are represented which are between the picture plane and the eye. Such a position of objects is the rule and not the exception in perspectives used for surveying, when they are taken as representations not of the ground itself, but of a model of it reduced to the scale of the map. This convention will be found further on. Objects are even represented which are behind the observer, the origin of light, for instance, in the construction of shadows, but this is merely a geometrical conception to which the usual definition of a perspective does not apply.

The "*ground line*" is the intersection of the ground and picture planes (*XY*, Fig. 59).

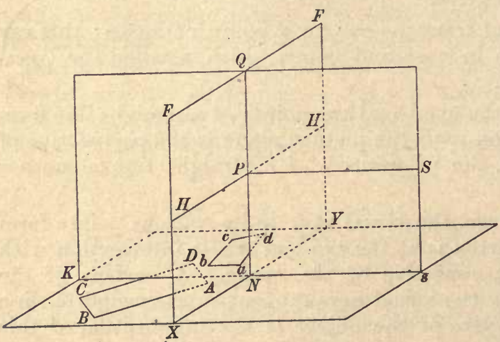

Fig. 59

The "*station*" is the point supposed to be occupied by the eye of the observer. (*S*. Fig. 59).

The "*foot of the station*" is the point where the vertical of the station pierces the ground plane. (*s*. Fig. 59).

The "*principal point*" is the foot of the perpendicular drawn from the station to the picture plane; it is shown in *P*, Fig. 59.

The "*distance line*" is the line between the station and the principal point (*SP*, Fig. 59). Its length is the distance from the station or from the foot of the station to the picture plane.

The "*horizon plane*" is the horizontal plane passing through the station. It contains the distance line and cuts the picture plane on a horizontal line passing through the principal point and called "*horizon line*" (*HH*, Fig. 59). The distance between the horizon line or the principal point and the ground line is equal to the altitude of the station.

The "*principal plane*" is the vertical plane perpendicular to the picture plane and passing through the station (SNQ, Fig. 59). It contains the foot of the station, the principal point and the distance line.

The "*principal line*" is the intersection of the principal and picture planes (QN, Fig. 59). It is perpendicular to the ground and horizon lines and intersects the latter at the principal point.

A "*front plane*" is a plane parallel to the picture plane.

A "*front line*" is any line contained in a front plane, therefore any line parallel to the picture plane.

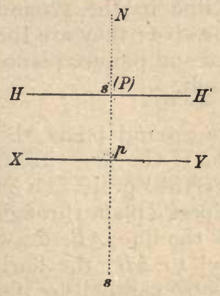

Fig. 60

In Fig. 60 these points, lines and planes are represented by their orthogonal projections; the ground plane is taken for horizontal plane and the picture plane for vertical plane; ss' is the station, s the foot of the station, s' or P the principal point, HH' the horizon line, sp, s' the distance line and spN the principal plane.

46. Perspective of a Point in the Ground Plane.—Let XYs, (Fig. 61), be the ground plane, XYN the picture plane, S the station and M a point in the ground plane. The perspective of M on the picture plane is the point where the straight line SM pierces the picture plane, that is the vertical trace of SM, M being the horizontal trace. Thus we have the first relation between a point of the ground plane and its perspective; they are the traces of the visual ray on the ground and picture planes respectively.

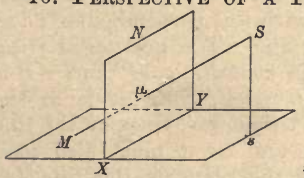

Fig. 61

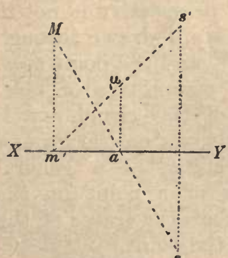

Fig. 62

Fig. 62 represents in orthogonal projection the construction of Fig. 61; ss' is the station, M the point of the ground plane sa, $s'm'$ the visual ray and μ the perspective of M. The points M and μ are the traces of sa, $s'm'$.

47. Perspective of a line in the ground plane.—It has been shown in § 44 that the perspective of a straight line is the intersection with the picture plane of the plane containing the station and the given line.

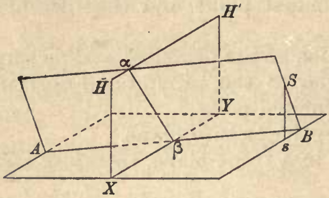

Fig. 63

Draw a plane through the straight line AB and the station S (Fig. 63). The intersection $\alpha\beta$ of this plane with the picture plane is the perspective of AB. Thus we have this relation between a straight line in the ground plane and its perspective; they are the traces on the ground and picture planes of the plane containing the station and the line itself.

In orthogonal projection, the line being in the ground plane, the horizontal projection is the line itself, AB (Fig. 64); the vertical projection is the ground line. To pass a plane through the station sP and the line AB, draw through sP a parallel to AB; the horizontal projection is a parallel to AB through s, and the vertical projection a parallel through P to the ground line. The vertical trace is at c', the intersection of $c'P$ with the perpendicular $c'c$ to the ground line. The horizontal trace of the plane containing sP and AB is the line AB itself, since it is in the ground plane. The vertical trace passes through c', trace of the line sc, Pc', which is contained in the plane; and as it must also pass through A, therefore the vertical trace of the plane is the line Ac'. Hence Ac' is the perspective of AB.

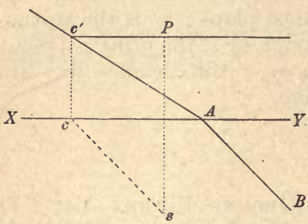

Fig. 64

48. Perspective of a point not in the ground plane.—The construction given in § 46 does not change when the point to be placed in perspective is not in the ground plane. A line is still drawn through the station sP, (Fig. 65), and the point mm'. The vertical trace μ is the perspective of mm'.

The horizontal trace a of the visual ray is the perspective of the point mm' on the ground plane; hence it may be stated as a general rule, that the perspectives of a point on the ground and picture planes are the traces of the line joining the station to the point.

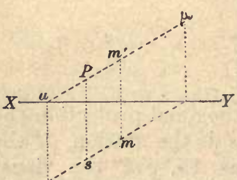

Fig. 65

49. Perspective of a line not in the ground plane.—Let *ab*, *a′b′*, (Fig. 66), be a line not in the ground plane: to obtain its perspective, a plane must be passed through the station *sP* and the line *ab*, *a′b′*; the intersection of this plane with the picture plane, that is the vertical trace of the plane, is the perspective of the line.

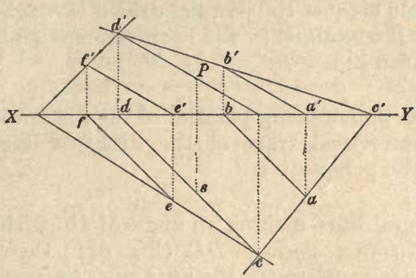

Fig. 66

Through *sP*, draw a parallel *sd*, *Pd′* to *ab*, *a′b′*; both lines are contained in the plane to be drawn, therefore the traces of the plane are the lines *ac*, *d′b′* joining the traces of same denomination of the parallels; and *d′b′*, the vertical trace of the plane, is the perspective of *ab*, *a′b′*.

Let us now consider another line, *ef*, *e′f′* parallel to *ab*, *a′b′*; the plane passing through *ef*, *e′f′* and the station *sP* must again contain the parallel *sd*, *Pd′*, through the station; therefore the vertical trace of the plane, which is the perspective of *ef*, *e′f′*, is the line *f′d′* joining the vertical traces of the two parallels. Hence the perspective of any line parallel to *ab*, *a′b′* will pass through the point *d′*. This result could be foreseen, because when a system of parallels has to be placed in perspective, all the planes serving to project them on the picture plane have a common line of intersection, parallel to the general direction of the system and passing through the station. Its trace on the picture plane must therefore be the common point of intersection of the perspectives. This point is called the "*Vanishing point*" of the parallel lines, because it represents the parts of the lines which are at infinity; their perspective ends or vanishes at that point.

The horizontal traces of the planes are the perspectives of the parallel lines on the ground plane. Like the perspectives of the picture plane, they all meet in a common point, which is the horizontal trace of the parallel line through the station; it is the vanishing point of the perspectives of the ground plane. Therefore, it is seen that when a plane is drawn through the station and a line in space, the traces of the plane on the picture and ground planes are the perspectives of the line on those planes.

50. Positions of the vanishing point.—A horizontal line has its vanishing point on the horizon line because the parallel drawn through the station, being horizontal, is all contained in the horizon plane and has its vertical trace on the horizon line.

Perpendiculars to the picture plane, being parallel to the distance line, have for vanishing point the vertical trace of the distance line which is the principal point of the perspective.

The vanishing points of horizontal lines making an angle of 45° with the distance line are called "*distance points*," DD, (Fig. 67); their distance from the principal point is equal to the distance line, because a horizontal line inclined at 45° to SP, forms an isosceles triangle SPD in which $SP = PD$.

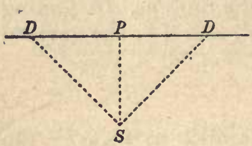

Fig. 67

Lines in the principal plane have their vanishing point on the principal line. Two of these lines form angles of 45° with the distance line, one above and the other below the horizon. Their vanishing points are known as "*upper and lower distance points*"; they are also at the same distance from the principal point as the station.

Lines parallel to the picture plane have no vanishing point. It will be shown later on that their perspectives are parallel to the lines themselves and do not meet.

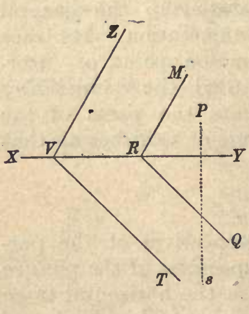

Fig. 68

51. Vanishing line.—Through the station sP (Fig. 68), pass a plane TVZ parallel to a given plane QRM. The vertical trace VZ contains the traces of all the lines drawn through the station parallel to QRM, it is therefore the locus of the vanishing points of parallels to the plane QRM. This trace VZ may be called the "*vanishing line*" of the plane QRM or of any other plane parallel to it (1).

The horizontal trace VT is in like manner the vanishing line of the perspectives of the ground plane.

(1) The term "vanishing line" is usually applied to the perspectives of parallel lines: admitting that the expression "vanishing point" is a proper one, the line VZ cannot be called otherwise than "vanishing line." This term is used here with that acceptation only.

52. Lines or figures in front planes.—The perspective of a straight line contained in a front plane is another straight line parallel to the first one. For the plane containing the station and the given line being cut by two parallel planes, the picture and front planes, the intersections are parallel lines. But these intersections are the line itself and its perspective, therefore the perspective is parallel to the given line.

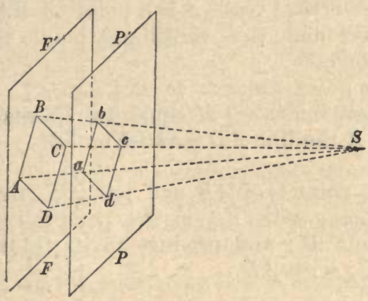

Fig. 69

Let S, (Fig. 69), be the station, PP', FF', the picture and front planes and $ABCD$ a polygon in the front plane, Join SA, SB, SC, SD; these lines intersect the picture plane at a,b,c,d, the polygon $abcd$ being the perspective of $ABCD$. The lines drawn from S form a pyramid of which the polygon of the front plane is the base and the perspective a section by a plane parallel to the base. It is shown in geometry that when a pyramid is cut by a plane parallel to the base, the section is a figure similar to the base. The front plane being parallel to the picture plane, the perspective must be similar to the original figure.

It follows that a curve in the front plane is represented by a similar curve in perspective, because such a line can be assimilated to a polygon with a great number of sides.

When the front plane is beyond the picture plane, as in Fig. 69, the perspective is smaller than the original figure; it is larger when the front plane is between the station and the picture plane, but in either case it is an exact representation of the figure itself, on a different scale. This scale, or the proportion between the perspective and the original figure is called the "*scale of the front plane.*" It is the ratio between the distance line and the distance from the station to the front plane.

A straight line parallel to the picture plane is contained in a front plane and is represented in perspective by a line parallel to itself; therefore parallel lines, which are also parallel to the picture plane have parallel lines for perspectives and have no vanishing point. The parallel to the given lines passing through the station, being parallel to the picture plane, has no trace on it.

Vertical lines are parallel to the picture plane and appear in perspective as parallels to the principal line.

Horizontal lines parallel to the picture plane are in perspective parallel to the horizon line.

53. Measuring lines and measuring points.—Let PP', (Fig. 70), be the picture plane, S the station and AB a straight line piercing the picture plane at A. Through S, draw the parallel SV to AB: V is the vanishing point of AB whose perspective is VA, since the vertical trace A is a point of the perspective and the vanishing point is another one.

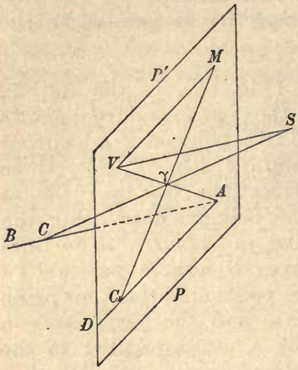

Fig. 70

Through V, draw VM equal to VS and through A the line AD parallel to VM.

Take a point C of AB and join CS, the intersection γ with VA is the perspective of C. Join $M\gamma$ and produce to its intersection C_1 with AD.

VS and AB being parallel the triangles $V\gamma S$ and $A\gamma C$ give the proportion:

$$\frac{VS}{AC} = \frac{V\gamma}{A\gamma} \qquad (1)$$

The triangles $VM\gamma$ and $AC_1\gamma$ are also similar, VM being parallel to AC_1, therefore:

$$\frac{V\gamma}{A\gamma} = \frac{VM}{AC_1} \qquad (2)$$

Hence from (1) and (2):

$$\frac{VS}{AC} = \frac{VM}{AC_1}$$

But by construction

$$VM = VS$$

therefore

$$AC = AC_1$$

The line AC, represented in perspective at $A\gamma$, is equal to the line AC_1.

Fig. 71 shows the picture plane with the same letters as in Fig. 70. The part of the line seen in perspective at $A\gamma$ is equal to AC_1. On AC_1 take another point D, join to M, and call δ the intersection with VA. The line seen in $A\delta$ is equal to AD, therefore the part seen in $\gamma\delta$ is equal to C_1D.

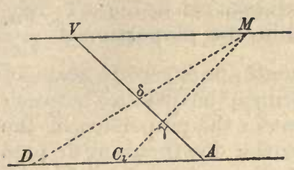

Fig. 71

The line AD is called the "*measuring line*" of AV, because it serves to measure the length of the line in space corresponding to any portion of its perspective AV; M is the "*measuring point.*"

VM was not drawn in any particular direction, therefore the direction of the measuring line, parallel to VM, is indeterminate. It is usual to make it parallel to the horizon line.

The position of the measuring point depends only on the vanishing point; therefore the same measuring point may serve for all lines parallel to the same direction.

The same measuring line will serve for all lines having their vertical traces on it. Should the line VM be drawn parallel to the vertical trace of a plane, this trace would be a measuring line for all lines contained in the plane.

If the measuring line is taken parallel to the horizon the measuring point of any horizontal line is on the horizon line, since the vanishing point is on that line. All lines in the same horizontal plane have then for measuring line the vertical trace of the plane, and lines in the ground plane have the ground line.

There is no measuring line or point for lines in a front plane, because they have no vertical traces or vanishing points; the scale of the front plane has to be employed when the length of such a line is wanted.

The distance points are measuring points for lines parallel to the distance line.

54. Reduction of a perspective to scale.—Hitherto it has been assumed that in the constructions, the real dimensions of the figures were employed. It would be quite impracticable to do so in the generality of cases. The dimensions must be reduced to a certain scale in order not to exceed the limits of the paper.

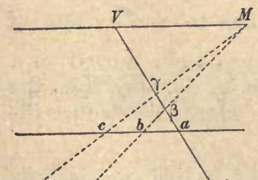

Fig. 72

By changing the position of the measuring line reduced distances can be used. Let V, M, and AC (Fig. 72) be the vanishing and measuring points and the measuring line of the perspective AV. The part of the line seen in $\beta\gamma$ is equal to BC. Through a point a of AV draw the parallel ac to AC and let us use it as a measuring line; the length corresponding to $\beta\gamma$ is bc, and we have the proportion:

$$\frac{bc}{BC} = \frac{Va}{VA}$$

Thus the lengths obtained are all reduced in the proportion of $\frac{Va.}{VA}$ Therefore, in order to obtain at once the length, on a certain scale, of a line seen in perspective, it is sufficient to reduce the distance between the measuring line and the vanishing point in the proportion of the scale to be employed. Thus, if Va be made the one-thousandth part of VA, the distances will be obtained on a scale of $\frac{1}{1000}$. M is the measuring point, and ac the measuring line, of a line having V for vanishing point and a for trace on the picture plane; the new line is therefore parallel to the line joining V to the station and to the original line seen in perspective, but its distance from the station has been reduced to the scale adopted.

Hence, to obtain the length reduced to scale of a line seen in perspective, reduce to scale the distance of the line from the station, moving it parallel to itself in the plane containing the station.

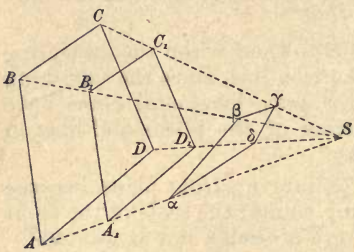

Fig. 73

The same conclusion is otherwise arrived at in a more direct manner. A figure $ABCD$ (Fig. 73) forms, with the visual rays joining it to the station, a pyramid, the intersection of which by the picture plane is the perspective $\alpha\beta\gamma\delta$.

Let the pyramid be cut by a plane parallel to the base $ABCD$; the intersection $A_1B_1C_1D_1$ is similar to $ABCD$, the proportion being $\frac{SA_1}{SA}$. The lines A_1B_1, B_1C_1 ...... measured by means of their perspectives $\alpha\beta$, $\beta\gamma$ ...... are therefore the lines AB, BC ...... reduced to the scale $\frac{SA_1}{SA}$. The same demonstration applies to any system of figures, whenever every point of the system has been moved in a straight line towards the station, so as to reduce its distance from the station in the proportion of the scale given. Hence we deduce the following important rule:—

To lay off dimensions reduced to scale or to measure them from a perspective, assume that the system formed by the station and the original figures or objects had been reduced to scale when the perspective was executed.

55. To place in perspective a point of the ground plane.—1st. By means of the principal point and a distance point.

Let M, (Fig. 74), be the point, XY the ground line, P and D the principal and distance points, the picture plane being revolved upon the horizontal plane. Draw MA at an angle of 45° and MB perpendicular to the ground line. The perspective of AM is the line AD joining the trace on the picture plane to the distance point. The perspective of MB, vanishing at the principal point is PB; therefore the perspective of M is at μ.

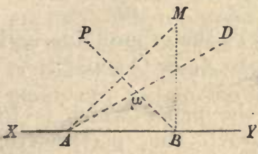

Fig. 74

2nd. By means of the distance of the point from the ground line.

Draw MB perpendicular to XY and take AB equal to MB; join AD and PB.

3rd. By means of the station and principal point.

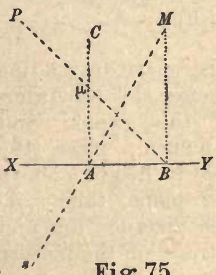

Fig. 75

Join the foot of the station s (Fig. 75) to the point M. The line sM is the horizontal trace of the vertical plane containing M and the station, which plane cuts the picture plane on a line AC perpendicular to XY. From M draw the perpendicular MB to the ground line; it is represented in perspective by PB, therefore μ (intersection of AC and PB) is the perspective of M.

4th. By means of the projection on the principal plane.

Revolve the principal plane around its trace sp (Fig. 76) upon the ground plane; the station will come in S_1 on a perpendicular to sp, sS_1 being equal to the altitude of the station. Draw Mm' perpendicular to sm' and join S_1m'; it is the projection on the principal plane of the visual ray from the station to the point M, and its intersection μ_1 with pY is the projection of the perspective of M revolved upon the ground plane. Join sM and at m erect a perpendicular $m\mu$ to the ground line; the perspective of M is on that perpendicular at a distance $m\mu$ equal to $p\mu_1$.

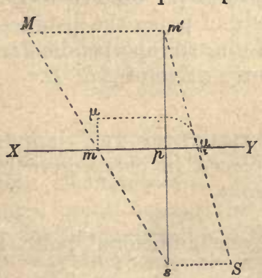

Fig. 76

When a great number of points have to be placed in perspective, this last method is very convenient. In practice the perspective is not constructed on the ground plan itself, as the operations would become confused; the plan and perspective are kept separate.

Let $ABCD$, (Fig. 77), be the ground plan, $X_2 Y_2$ the ground line, and $s_2 p_2$ the trace of the principal plane. Join s_2 to A, B, C and D.

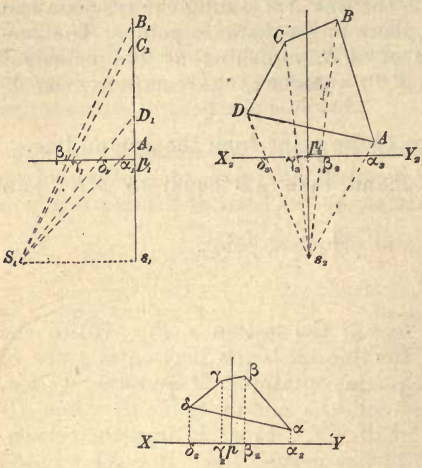

Fig. 77

On the paper which is to serve for the perspective, draw the ground line XY and take a point p as intersection of the principal plane. Take on the edge of a piece of paper the distances from p_2 to α_3, β_3, γ_3, δ_3, and carry them on XY in α_2, β_2, γ_2, δ_2; at the last mentioned points erect perpendiculars to the ground line.

At another place draw a line $s_1 B_1$ to represent the intersection of the ground and principal planes; place the station S_1 at its height h above the ground plane, take $s_1 p_1$ equal to the distance line and draw the trace of the picture plane, $p_1 \beta_1$ perpendicular to $s_1 B_1$.

On the edge of a piece of paper, take the distances of A, B, C, D, from the ground line $X_2 Y_2$ and carry them on $p_1 B_1$. Join S_1 to A_1, B_1, C_1, D_1. Again take on the edge of a piece of paper the distances of α_1, β_1, γ_1, δ_1 from p_1, and lay them on the perpendiculars $\alpha_2\alpha$, $\beta_2\beta$, $\gamma_2\gamma$, $\delta_2\delta$. This gives $\alpha\beta\gamma\delta$ as the perspective of $ABCD$.

56. To place in perspective a line or figure of the ground plane.—A line of the ground plane may be placed in perspective by determining the perspectives of two of its points.

When the vanishing point is known, only one additional point is required to define the perspective.

With a figure composed of straight lines, the perspectives of the points of intersection are fixed and joined together by straight lines.

The perspective of a curve is found from the perspectives of a sufficient number of points or by tangents to the curve.

57. To PLACE IN PERSPECTIVE A POINT OUTSIDE OF THE GROUND PLANE.—When a point is not in the ground plane, the perspective of its horizontal projection is first found: the height of the point above or below the ground plane is next reduced to the scale of the front plane and laid on the vertical of the perspective previously found.

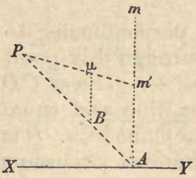

Fig. 78

Let m, (Fig. 78), be the projection of the point on the ground plane and B the perspective of m, obtained as in § 55. From m let fall the perpendicular mA on XY and take Am' equal to the height of the point. Join m' to the principal point, P; Pm' is the perspective of the perpendicular through M to the picture plane, therefore the perspective of M must be on Pm'. But the given point M is on the vertical line passing through m whose perspective is the perpendicular $B\mu$ to the ground line, therefore the perspective of the point M is at the intersection μ of the two lines.

Comparing Fig. 78 with Fig. 65, § 48, it will be seen that the construction is precisely the same, although made on different principles.

58. To PLACE IN PERSPECTIVE A LINE OUTSIDE OF THE GROUND PLANE.—When a line is in a horizontal plane, that plane may be taken temporarily as ground plane and changed when the perspective has been obtained.

If in any other plane, the perspective may be found by means of the vanishing point and horizontal trace. The latter is placed in perspective as explained in § 55, and joined to the vanishing point.

For lines in front planes, one point of the line is placed in perspective and through it, a parallel to the line is drawn.

59. THE DISTANCE LINE IS AN AXIS OF SYMMETRY OF THE PERSPECTIVE.—A perspective is symmetrical with reference to the distance line, all points of the picture plane at the same distance from the principal point having the same geometrical properties. Therefore, any plane perpendicular to the picture plane may be taken as ground plane, or any line through the principal point as horizon line. So when figures are contained in a plane perpendicular to the picture plane, their perspectives can be obtained by taking the plane of the figures for ground plane and its vertical trace for ground line.

60. Given the heights of two points and their perspectives, to find the vanishing point and trace on picture plane of the line joining the given points.—Let *HH* and *P*, (Fig. 79), be the horizon line and principal point, α and β two points of the perspective. Draw *EF* parallel to the horizon line at a distance equal to the height of α: it is the trace of the horizontal plane containing the point of space seen in α.

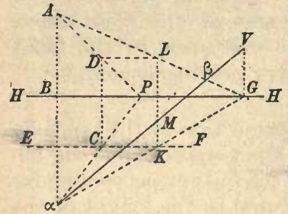

Fig. 79

The perspective of the perpendicular to the picture plane passing through this point is $P\alpha$: its vertical trace is C. Draw CD perpendicular to the horizon line and equal to the height of β above the plane of α. D is a point of the picture plane at the same height as β, and PD is the perspective of the perpendicular to the picture plane passing through D; PD is in the same vertical plane as PC and if produced will meet the vertical of α seen in perspective at αB. The point of intersection A is at the same height as D and β, therefore $A\beta$ is a horizontal line and its vanishing point is on the horizon line at G. But $A\beta$ and $\alpha\beta$ are in the same vertical plane having for vertical trace the perpendicular GV to the horizon line, therefore the vanishing point of $\alpha\beta$ is at its intersection V with GV.

To find the trace, draw through D the parallel DL to the horizon line: it is the trace on the picture plane of the horizontal plane containing AG, and the trace of AG is at its intersection L with DL. But AG and αV being in the same vertical plane, the trace of αV is in M, on the perpendicular LM to the horizon line.

αG is a horizontal line also in the same vertical plane as AG and αV: consequently its trace is K, on LM produced. But αG is in the horizontal plane whose trace is EF, therefore K is the intersection of αG and EF.

61. To find the intersections of a vertical line by a series of horizontal planes.—Let HH' and FG, (Fig. 80), be the horizon and principal lines of a perspective, μ a point of the perspective $\eta\theta$ of a vertical line, of which the altitude above or below the station is known. Take PM equal to this altitude: M is a point of the picture plane at the same altitude as μ. Join μM: it is the perspective of

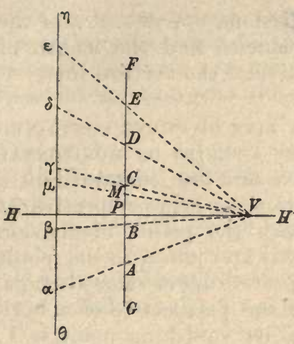

Fig. 80

a horizontal line having its trace in M and its vanishing point at V. Mark on FG the intersections A, B, C, D, E of the horizontal planes, join to V and produce VA, VB, VC, VD, VE to $\eta\theta$; these lines are the perspectives of horizontal lines parallel to μM and contained in the horizontal planes. Their intersections α, β, γ, δ, ε, with the perspective of the vertical line $\eta\theta$ are the points required.

This construction is employed for determining the intersections of a vertical line by contour planes: the equidistance is marked on the edge of a piece of paper which is pinned along GF so that P corresponds to the altitude of the station. A straight edge is placed on μ and the point of same height of the equidistance scale, then a pin is planted at V and the straight edge moved through each of the points A, B, ... E, always keeping it in contact with the pin.

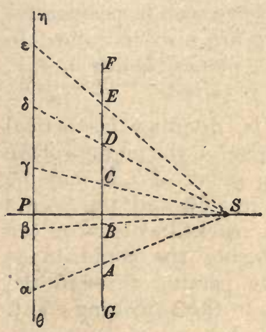

Fig. 81

Another solution consists in projecting the vertical line and its perspective on the principal plane.

Let SP, Fig. 81, be the distance line, $\eta\theta$ the principal line and FG the intersection of the front plane containing the vertical line, by the principal plane. Mark on FG the intersections of the horizontal planes, join to S and produce to $\eta\theta$; the intersections are the projections on the principal plane of the points required.

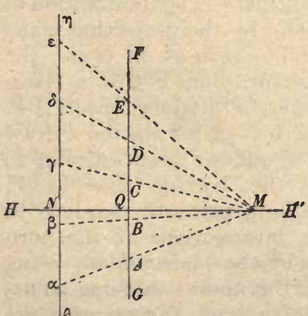

Fig. 82

In practice, the construction is made on the perspective: $\eta\theta$ (Fig. 82), being the perspective of the vertical line, NM is taken on the horizon line equal to the distance line and NQ equal to the distance of the vertical line from the picture plane. At Q a perpendicular is erected to HH' and the equidistance scale pinned alongside, so that Q shall correspond to the altitude of the station. The construction is completed as in Fig. 81.

Contour planes being equidistant, the divisions $\alpha\beta$, $\gamma\beta$ . . . . . of the perspective are equal: it is therefore sufficient to find the length of one division and to carry it on the perspective of the vertical line.

62. TO MARK ON THE PERSPECTIVE OF ANY LINE OR CURVE CONTAINED IN A VERTICAL PLANE, THE INTERSECTIONS BY A SERIES OF HORIZONTAL PLANES.—Let $\mu\delta$ (Fig. 83), be the perspective of a line contained in a vertical plane: that plane contains the vertical seen in perspective at δM, perpendicular to the horizon line. Mark the points of division A,B,C, of δM by the horizontal planes, (§ 61) and join μ to the point of the perspective δM of same altitude. This being the perspective of a horizontal line, its vanishing point is V. Join V to A,B,C: these lines are the perspectives of parallels to $M\mu$, therefore they are in the plane $\mu M\delta$ and intersect the curve seen in perspective at $\mu\delta$, but they are also contained in the horizontal planes, hence α, β, γ, are the points required.

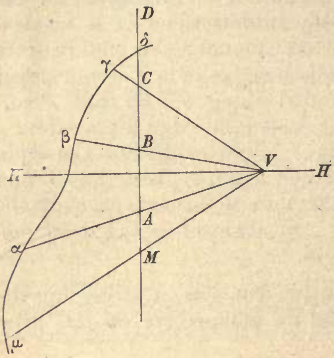

Fig. 83

Instead of first dividing the vertical line δM, the trace on the picture plane and vanishing point of μM may be determined as in § 60 and the points of intersection marked at once on the line $\mu\delta$ by placing the equidistance scale on the perpendicular to the horizon line passing through the vertical trace and joining the points of division to the vanishing point.

When the horizontal projection of the line is known, the vanishing point and trace are obtained as follows: let $\alpha\beta$, (Fig. 84), be the perspective of the line, ab its horizontal projection, HH' the horizon and XY the ground line. The intersection of the ground plane by the projecting plane containing the line seen at $\alpha\beta$ is ab; the trace on the picture plane of this intersection is at E, where ab produced meets XY.

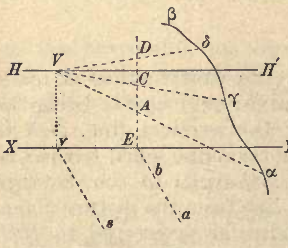

Fig. 84

Through the foot of the station s, draw sv parallel to ab and vV perpendicular to XY, meeting the horizon line in V. V is the vanishing point of parallels to ab. But the intersections of the horizontal planes by the plane of $\alpha\beta$, being parallel to ab, V is their vanishing point; and since they are all in the same vertical plane, their traces are on the vertical ED of the picture plane. Hence the equidistance

scale is to be placed along ED, taking care that the point E of the scale corresponds to the altitude of the ground plane; the divisions of the scale are joined to the vanishing point and produced to their intersection with the perspective.

63. To mark on the perspective the intersections of a plane, line, or curve, by a series of horizontal planes.—The intersections of a plane by a series of horizontal planes are horizontal lines parallel to the trace, on the ground plane, of the plane intersected; the vanishing point of these lines is the point of intersection of the horizon line by a parallel to this trace drawn through the station.

Let $\alpha\gamma$ (Fig. 85) be the perspective of a line or curve in the plane POP', XY the ground line and HH' the horizon line. Through the foot of the station s draw sv parallel to OP, and erect the perpendicular vV to the ground line, meeting the horizon line in V: V is the vanishing point of horizontal lines in the plane POP', and, consequently, of the intersections of that plane by the horizontal planes. The traces of these lines on the picture plane are on OP' and the vertical distance between them is that of the horizontal planes: therefore place at O, on a perpendicular to the ground line, the distances of the horizontal planes or the scale of equidistance, draw parallels to the ground line through the divisions A, B, C of the scale, and join A', B', C' to the vanishing point. These lines are the perspectives of the intersections of the plane POP' by the horizontal planes.

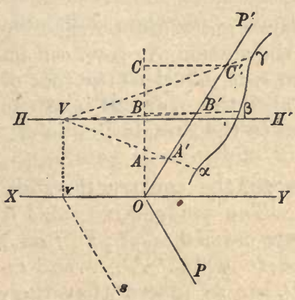

Fig. 85

64. Intersections of a prism, pyramid, or conic surface, by a series of horizontal planes. — The intersections of a prism or pyramid by a series of horizontal planes can be drawn on the perspective by determining the intersections of the edges of the prism or pyramid by the planes and joining the corresponding points by straight lines.

A similar process can be applied to a conic surface by using generating lines instead of edges, and also by employing tangents to the intersections, parallel to the tangents drawn to the curve of the ground plane forming the base of the cone.

65. To place a point of the ground plane by means of its perspective.—To restore a figure by means of its perspective is the converse of perspective. Let us consider first the case of a point of the ground plane; its place can be found by inverting any of the constructions given in § 55.

For instance, in Fig. 74, the perspective μ of the point is joined to the principal and distance points, P and D. At B a perpendicular BM is erected to the ground line and from A a line AM is drawn at an angle of 45° with the ground line. M is the point of the ground plane.

In Fig. 75, join $P\mu$ and draw μA and BM perpendicular to the ground line. Join sA and produce to intersection with BM.

In Fig. 76, $p\mu_1$ is taken equal to the distance μm of the perspective μ from the ground line and μ_1 is joined to the station S_1 revolved on the ground plane. The foot of the station s, is joined to the foot of the perpendicular μm to the ground line and the point M is at the intersection of sm produced with the parallel to the ground line $m'M$.

When a number of points have to be placed, the constructions are made as in Fig. 77, but in inverse order. The perspective $\alpha\beta\gamma\delta$ is given; the distances $p\alpha_2$, $p\beta_2$, ... are carried on X_2Y_2; $\alpha\alpha_2$, $\beta\beta_2$ .... on $p_1\beta_1$. Then s_2 is joined to $\alpha_3\beta_3$ .... and on these lines produced the points A, B, C, D, are so placed that their distances from the ground line are equal to p_1A_1, p_1B_1, p_1C_1, p_1D_1.

66. To place a line on the ground plane by means of its perspective.—The trace of the line on the picture plane is the point where its perspective intersects the ground line: this point is common to the perspective and to the line (β, Fig. 86).

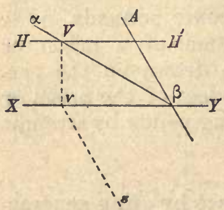

Fig. 86

The vanishing point, V, is the intersection of the perspective by the horizon line; it gives the direction of the line of the ground plane.

From V, let fall the perpendicular Vv to XY and join vs. Through β draw βA parallel to sv; it is the required line.

In the case of a front line, a point of the line is fixed by one of the methods of § 65 and a parallel to the ground line drawn through the point.

67. To draw a figure on the ground plane by means of its perspective.—A figure of the ground plane may be constructed by means of its perspective as described in § 65, each of the summits of the figure being determined separately.

It may also be constructed by determining each of the lines forming the figure, as in § 66.

An irregular figure is inclosed between straight lines and drawn at sight.

A convenient method is that known as the "*method of squares.*" The ground plane is divided into squares by lines parallel and perpendicular to the ground line; the network of squares is projected on the perspective and the figure drawn at sight in the corresponding squares.

To construct the perspective of the squares, the distances of the parallel lines are marked on the ground line in A, B, C, D, E, (Fig. 87), the perspectives of the perpendiculars to the ground line are obtained by joining these points to the principal point P.

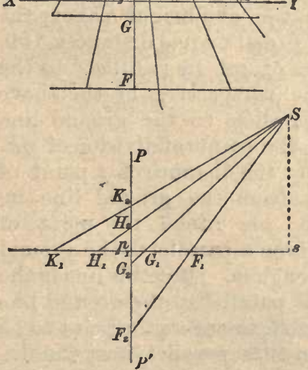

Fig. 87

The principal plane is next plotted separately, sK_1 being the trace of the ground plane, S the station and PP' the trace of the picture plane. Mark the intersections F_1, G_1, H_1, K_1, of sK_1 by the lines parallel to the ground line, join to S and carry to Pp the distances from p_1 to F_2, G_2, H_2, K_2: through the points so obtained, F, G, H, K, draw parallels to the ground line, which will complete the perspective of the squares.

It is not necessary that the sides of the squares be parallel or perpendicular to the ground line. Any other direction may be adopted, as for instance, north and south, and east and west in the case of topographical perspectives.

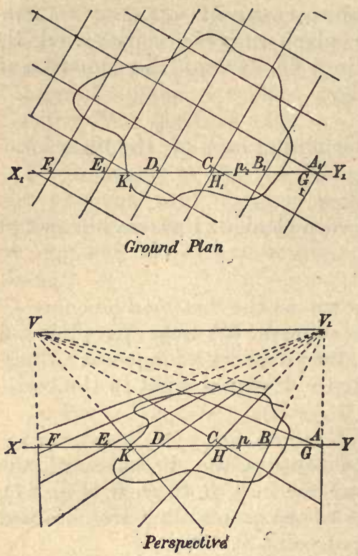

Fig. 88

The vanishing points V and V_1, (Fig. 88), of these lines are found as usual by drawing through the station parallels to their directions until they meet the horizon line.

The points of intersection with the ground line of the north and south lines, which will be supposed to vanish at V_1, are taken from the ground plan, carried to the ground line of the perspective, in A, B, C, D, E, F, and joined to V_1 : this gives the perspective of one set of parallel lines. The other set is obtained by a similar process, carrying the points G_1, H_1, K_1 from the ground plan to the perspective and joining to the vanishing point V.

The squares must be made small enough to guide the draughtsman accurately in transferring the figure from the perspective to the ground plan.

68. Vanishing scale.—The direction of a point of the ground plane is easy to find : it is sufficient to join the foot of the station to the projection of the perspective on the ground line. Were the distance of the point determined, it could be located at once. This is done by means of the "*vanishing scale*".

Fig. 89 represents the principal plane : Pp and pA are the traces of the picture and ground planes and S the station at a height h above the ground plane. On pA and on each side of p, mark equal distances, 100, 200, etc. : they represent the intersections of pA by parallels to the ground line. Join these points to S : the perspectives of the above parallels are parallels to the ground line passing through the points of division of pP. Suppose now that the distance of a point of the perspective from the ground line be found equal to pm : then the point of the perspective is on a parallel to the ground line passing through m. But this line is the perspective of a parallel to the ground line passing through M, therefore the point to be found, being on that parallel, is at the distance pM from the ground line. M and m corresponding to the same divisions of the scales pP and pA, the distance of the point is obtained at once by reading the division of pA corresponding to m.

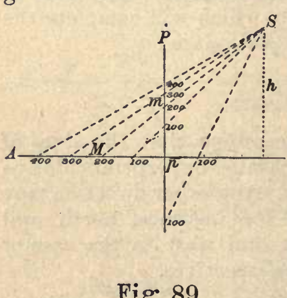

Fig. 89

The scale constructed as above on pP is called a "*vanishing scale.*"

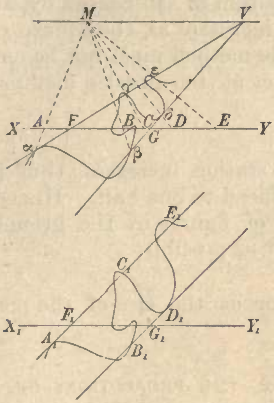

Fig. 90

69. Use of the measuring line.—Sometimes the greater part of an irregular figure in a horizontal plane, may be inclosed between two parallel lines, as in Fig. 90. A point V is taken on the horizon line such that two lines drawn from it inclose the figure $\alpha\,\beta\,\gamma\,\delta\,\varepsilon$ as well as possible. These lines are the perspectives of two parallel lines in the ground plane and their vanishing point is V. Draw these parallels on the ground plan in $A_1\,E_1$ and $B_1\,D_1$ and place on the perspective the measuring point by taking VM equal to the distance of V from the station. The measuring line is the ground line XY. Find the distances from F and G to the points of the parallels corresponding to various points of the irregular figure and transfer them in A_1, B_1, C_1, D_1, E_1, to the ground plan. Draw the intermediate parts of the figure at sight.

Should two parallel lines prove insufficient, the number can be increased.

The method of squares, the vanishing scale and the measuring line can be employed for finding the perspective from the ground plan. The operations are the converse of the preceding ones and require no further explanation.

70. Precision of the method.—Let Ss and M, (Fig. 91) represent the vertical passing through the station and a point of the ground plane, Sm and μA, the traces of the horizon and picture planes and μ the perspective of M. Draw Mm perpendicular to Sm: it is the height, h, of the station above the ground plane.

The similar triangles $SA\mu$ and SmM give:

$$\frac{Sm}{mM}=\frac{SA}{A\mu}$$

or

$$\frac{y}{h}=\frac{l}{x} \tag{1}$$

To find the effect on the distance y from the station to M, of an error dx in the perspective, equation (*1*) must be differentiated, considering x and y as variables; this gives:

$$dy=-\frac{y}{x}dx=-\frac{y^2}{hl}dx \tag{2}$$

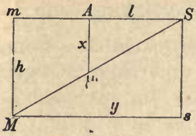

Fig. 91

So the error in the position of M caused by an error in the perspective increases as the square of the distance: therefore the method must not be employed for points or figures at too great a distance from the station.

The error decreases as the height of the station increases: thus if the height be doubled, the error will be reduced to one half. Hence, perspectives intended for the reproduction of figures in the ground plane should be taken from as great a height as possible.

The error decreases also as l increases, or as the size of the perspective increases.

71. To DETERMINE FROM THE PERSPECTIVE, THE PROJECTIONS OF A POINT NOT IN THE GROUND PLANE, BUT OF WHICH THE HEIGHT IS KNOWN. The perspective of a point is not sufficient to determine its position; other data must be furnished, such as the traces of a plane containing it, its distance, or its height above the ground plane.

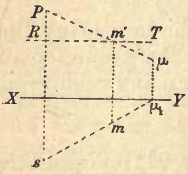

Fig. 92

If the height be known, draw a parallel RT, Fig. 92, to the ground line representing the trace on the picture plane of the horizontal plane containing the point. The projections of the visual ray joining the station to the point are $s\mu_1$, $P\mu$ (§ 47): it pierces the horizontal plane RT in m, m', and as the point to be found is in that plane and on the line $s\mu_1$, $P\mu$, it is the point of intersection, mm'.

The construction is not always possible. For instance RT may pass through P: this means that the point is in the horizon plane, in which case it cannot be located by means of its perspective.

$P\mu$ may coincide, or very nearly, with Ps, and the construction become impossible or uncertain. The visual ray joining the station to the point is then projected on the principal and ground planes instead of the picture and ground planes: the different steps are precisely the same in both methods.

72. To CONSTRUCT FROM ITS PERSPECTIVE A FIGURE IN ANY HORIZONTAL PLANE.—The methods given in §65, 66 and 67 apply to figures in any horizontal plane, by using the planes of the figures as ground planes; all that is required being to shift the ground line on the perspective to its proper position.

73. To find the traces and vanishing point of a line given by its horizontal projection and perspective.—Before proceeding to consider figures in various planes, it is necessary to show how the plane of a figure and the traces of straight lines can be determined.

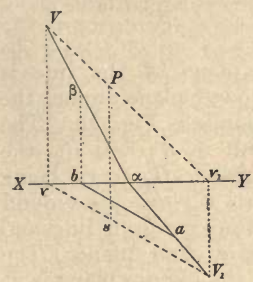

Fig. 93

Let $a\beta$ and ab (Fig. 93), represent the perspective and horizontal projection of a line. At b draw a perpendicular to the ground line: the trace on the picture plane must be on that perpendicular and also on $a\beta$, therefore it is at their intersection β.

The vanishing point is the trace of a parallel to the line drawn through the station; the horizontal projection of this parallel is sv, drawn through the foot of the station parallel to ab, and its trace is on the perpendicular vV to the ground line. But this trace is the vanishing point of $a\beta$; therefore it is at the intersection of vV and $a\beta$ produced.

The vertical projection passes through the trace V and the principal point P; producing it to the intersection with XY and drawing the perpendicular $v_1 V_1$ to XY, the trace on the ground plane is found at V_1.

The line joining V_1 to a is the perspective on the ground plane of the given line (§ 49) whose trace is the intersection of aV_1 and ab.

The trace on the ground plane may also be found by revolving the projecting plane of ab around its vertical trace $b\beta$ (Fig. 94) upon the picture plane. Draw the horizon line Pd; the trace of the given line on the horizon plane is seen in γ on the perspective; its horizontal projection is at the intersection c of ab by the line joining the foot of the station to the foot f of the perpendicular γf to XY. When the projecting plane revolves, c describes the arc of circle cc_1, with b as centre: the point of the given line corresponding to γ moves in the horizon plane; therefore it comes in γ_1 on the horizon line, at the intersection with the perpendicular $c_1\gamma_1$ to XY. The revolved line is βa_1, and the revolved trace on the ground plane is a_1. Revolving a_1 back to ab, the trace is obtained in a.

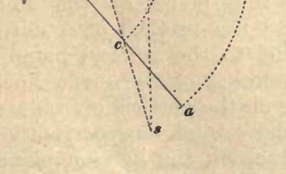

Fig. 94

The angle formed in a_1 by the revolved line and XY is the angle of the line with the ground plane.

A third method consists in determining from the perspective the heights of two points of the given line, as will be explained later on.

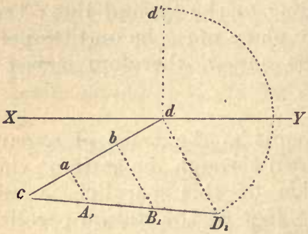

Fig. 95

The projecting plane of the line is revolved on the ground plane around the horizontal projection ab, Fig. 95. The points A and B fall in A_1 and B_1, the perpendiculars aA_1 and bB_1 to cd being the heights of A and B above the ground plane. A_1B_1 is the revolved line and c its trace on the ground plane. The revolved trace on the picture plane is at the intersection of A_1B_1 produced with the perpendicular dD_1 to cd; it is revolved back to the picture plane by drawing a perpendicular dd' to XY and describing an arc of circle with d as centre dD_1 as radius.

74. Given the slope of a line and the horizontal projection of one of its points, to find the horizontal projection and traces of the line.—Let a (Fig. 96), be the horizontal projection of a point of the line seen in perspective in $\alpha\beta$, s the foot of the station and XY the ground line. Join sa, produce to m and erect the perpendicular $m\alpha$ to XY; α and a are the perspective and projection of the same point, A, of the given line. Draw the horizon line HH': δ is the perspective of the trace of the given line on the horizon plane. Rotate the projecting plane of the line around the vertical of a until parallel to the vertical plane; the point A of the given line, being on the vertical of a, does not move, and its perspective

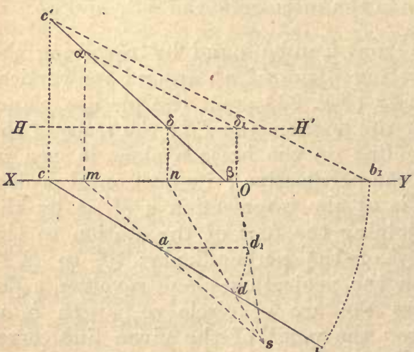

Fig. 96

remains in α. The perspective δ of the trace on the horizon plane moves on the horizon line: when the projecting plane is parallel to the vertical plane, the perspective of the rotated line is parallel to the line itself and may be drawn in $\alpha\delta_1$, since the angle $\alpha\delta_1H$ is given.

The trace of the projecting plane on the ground plane has come in ad_1 parallel to the ground line. The point d_1 of the horizontal projection corresponding to δ_1 of the perspective is obtained by drawing $\delta_1 O$ perpendicular to XY and joining sO. Rotating back the projecting plane to its original position, δ_1 comes in δ, the corresponding point of the horizontal projection being on the line sn joining the foot of the station to the foot of the perpendicular from δ to the ground line. But this corresponding point is the new position of d_1, and d_1 moves on an arc of circle with a as centre, therefore d_1 comes in d and da is the horizontal projection of the given line.

The vertical trace is found at c' by the usual construction : the vertical projection and horizontal trace may be determined as in § 73 or the triangle formed by cc', cb and the given line may be revolved around cc' on the vertical plane. The axis cc' does not move, cb falls on the ground line and the hypothenuse $c'b_1$, becomes parallel to $a\delta_1$. Revolving the triangle back to its original position, b_1 comes in b, which is the trace, on the ground plane, of the given line. Having now the two traces, the vertical projection can be drawn by the usual construction.

75. To FIND THE TRACES OF THE PLANE CONTAINING THREE GIVEN POINTS OR TWO GIVEN LINES.— Whether two lines or three points be given, the problem consisting in passing a plane through them is the same, and consists in finding the traces of the given lines or of those joining the given points. The traces of same denomination are joined by straight lines, which are the traces of the required plane.

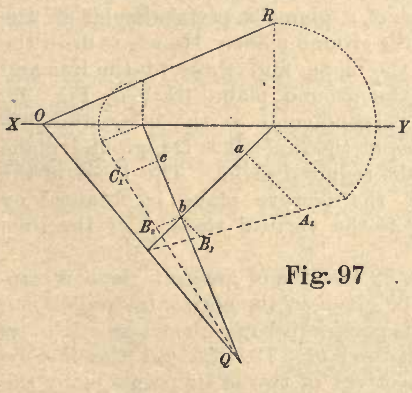

Fig. 97

The traces of the lines are obtained by any of the processes of § 60, 73 or 74.

In Fig. 97, the heights of the three points A, B and C are supposed to be known and the traces are determined by revolving the projecting planes on the ground plane around the horizontal projections ab and bc; (§ 73). QOR is the required plane.

Sometimes the traces of the plane are required on the picture and principal planes. Revolve the principal plane around its trace Rp, (Fig

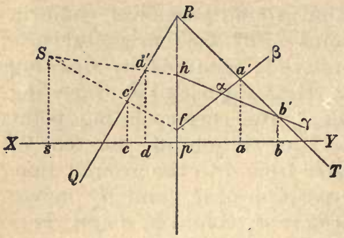

Fig. 98

98) on the picture plane, the front part of the principal plane turning to the left. The station comes in S.

Let α, β, γ, be the perspectives of three points A, B, C, of which the projections on the ground plane are given, a and c the traces on the picture and principal planes of the horizontal projection of the line AB, d and b those of the horizontal projection of AC.

Produce $\alpha\beta$ to the intersection f with the principal line; f is the perspective of the trace on the principal plane, of the line of space AB, therefore the trace on the revolved principal plane is on Sf. But the trace is on the vertical of c, therefore it is at c'. The trace of the other line is found in a similar manner at d' and the trace of the plane containing the two lines is $c'd'$.

The traces of the two lines AB and AC on the picture plane are obtained in a' and b' as in § 73, and being joined, give the trace of their plane on the picture plane. The result is the plane QRT.

76. Given the line of greatest slope, to find the traces of the plane.—The line of greatest slope of a plane is perpendicular to the trace on the ground plane. Hence, to draw the traces of the plane, find those of the line and through the ground plane trace a, Fig. 99, draw aQ perpendicular to the horizontal projection, ab, of the line: it is the ground plane trace of the required plane. The trace of the plane on the picture plane is obtained by joining Q to the vertical trace, β, of the line.

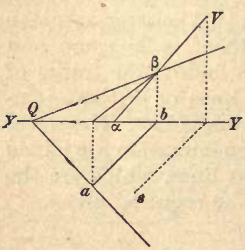

Fig. 99

In Fig. 99 the line of greatest slope is supposed to be given by its horizontal projection, ab, and its perspective $\alpha\beta$: the traces are found by the method of § 73. Should the line be known by the heights and perspectives of two of its points or by the heights and horizontal projections, or by its slope, the traces could be determined by the methods given in § 60, 73 and 74.

77. Change of ground plane.—A change in the ground plane does not produce any change in the points or lines of the ground plan: the traces of planes are displaced, but remain parallel to the original trace.

Fig. 100 shows the ground line moved from XY to X_1Y_1; the left hand figure contains the projections of a point, of a line and the traces of a plane before the change of ground plane.

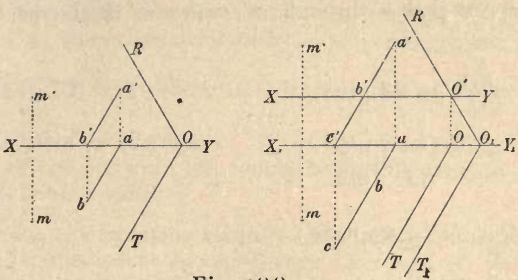

Fig. 100

In the first place, it may be observed that there is no change in the vertical plane, beyond moving the ground line from XY to X_1Y_1.

In the ground plane, the projections of the point m and of the line ab remain the same, but the trace of the line is now in c instead of b. The new trace is obtained by producing the vertical projection $a'b'$ across the old ground line XY to the new one, drawing the perpendicular $c'c$ and producing ab to meet $c'c$.

The trace of the plane has been moved from OT to O_1T_1. To find the new one, produce the vertical trace $O'R$ across the old ground line XY to the new one X_1Y_1, and through the point of intersection O_1 draw O_1T_1 parallel to OT.

78. To find the horizontal projection of a figure from its perspective when the figure is contained in a plane perpendicular to the principal plane.—Take for vertical plane of projection the principal plane and let QZ, (Fig. 101), be the trace of the plane containing the figure. Take for ground plane the horizontal plane passing through the point of intersection of QZ with the trace QR of the picture plane, XY being the ground line. Let S be the station, s the foot of the station, nn' a point of the given figure and mm' its perspective. The given plane, being perpendicular to the principal plane, the vertical projection of any point of the former is on the trace QZ. The picture plane RQT is perpendicular to both planes of projection, therefore the projections of any point of the picture plane are on its traces.

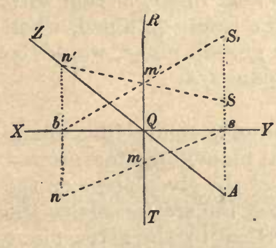

Fig. 101

Produce QZ to meet the vertical of the station in A and take SS_1 equal to sA, S_1 being above or below S according as A is below or

above s. Join S_1m' and produce it to meet the ground line in b : join Sn' and $n'b$. The line Sn' passes through m', since m' is the perspective of n'.

The similar triangles $n'm'Q$, $n'SA$ give :—

$$\frac{Qm'}{SA}=\frac{n'Q}{n'A}$$

From the triangles bQm', bsS_1, we have :

$$\frac{Qm'}{sS_1}=\frac{bQ}{bs}$$

But $$SA=sS_1$$

Therefore : $$\frac{n'Q}{n'A}=\frac{bQ}{bs}$$

or, $$\frac{n'Q}{QA}=\frac{bQ}{sQ}$$

Hence the triangles $n'bQ$, QsA are similar, as having one angle equal and the sides about it proportional, consequently bn' is parallel to sA or perpendicular to XY and the point n is the trace on the ground plane of the visual ray sn, S_1b. Were the eye placed in S_1, the point of the ground plane which would be found to correspond to mm' of the perspective would be the horizontal projection n of the point of the plane QZ. Should the new station S_1 be used in connection with the perspective of a figure in the plane QZ, the result obtained, when constructing the corresponding figure of the ground plane, would be the horizontal projection of the figure of the plane QZ.

Therefore to obtain the horizontal projection of a figure in a plane perpendicular to the principal plane, take for ground line the trace XY, (Fig. 102), of the given plane on the picture plane, find the height of the station above the point of intersection of its vertical by the given plane (§ 75) use it as height of the new station and draw the horizon line $H_1H'_1$ on the perspective at that height above the ground line. The figure constructed from the perspective by any of the methods of § 65, 66 or 67 will be the horizontal projection of the figure in space.

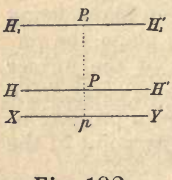

Fig. 102

It has been shown that the perspective is the same as if the horizontal projection had been seen from the station S_1 (Fig. 101) instead of observing the original figure from S; consequently, the precision of the result (§ 70) is increased in the proportion of $\frac{sS_1}{sS}$ by the inclination of the plane of the figure. Were the plane falling instead of rising in front of the observer, sS_1 would be smaller than sS and the precision would be decreased.

Hence a perspective taken for the purpose of constructing a figure in an inclined plane should always be taken in the direction of the rising plane; thus a river at the bottom of a sloping valley should be taken looking up the valley.

79. To find from its perspective the horizontal projection of a figure in a plane perpendicular to the picture plane.—The method of squares of § 67, can be applied to a figure in any inclined plane, by conceiving vertical planes containing the sides of the squares. The intersections of these planes by the inclined plane form a series of parallelograms corresponding to the squares of the ground plane.

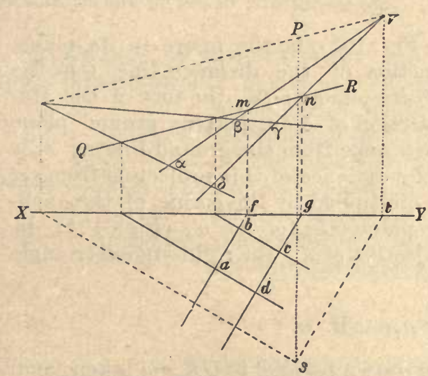

Fig. 103

Let QR (Fig. 103) be the trace on the picture plane of a plane perpendicular to it, XY the ground line, P the principal point, and $abcd$ one of the squares of the ground plan. The projecting planes of ab and cd cut the trace QR in m and n. Through the station, S, draw a parallel to the intersection of the projecting planes with the plane QR; the horizontal projection st is parallel to ab and cd; the vertical projection passes through P and is parallel to QR, since all lines in the plane QR are projected vertically on QR. At t draw the perpendicular tV to the ground line; V is the vanishing point of the intersections of the projecting planes with the plane QR and the lines Vm and Vn are the perspectives of these intersections. The distance mn can be carried on QR and as many parallels placed in perspective as necessary.

The same operation is repeated for ad and bc, and the figure $\alpha\beta\gamma\delta$ obtained on the perspective corresponds to the square $abcd$.

Another process consists in constructing the figure in the inclined plane by one of the methods of § 65, 66 or 67, using the plane of the figure as ground plane (§ 59).

Let QR, (Fig. 104) be the trace of the plane of the figure on the picture plane, HH' the horizon line and P the principal point.

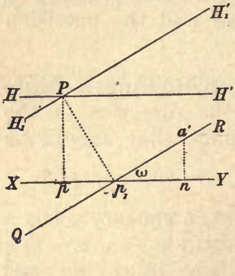

Fig. 104

To construct the figure in the plane QR, that line is taken as ground line: the new horizon line is a parallel $H_1H'_1$ to QR through the principal point. The height of the station is the distance between these two lines, Pp_1. The line which will appear as the projection of the principal line on the constructed figure is the perpendicular to the picture plane at p_1. On the true ground plane, the distance between the two projections of the principal line is equal to pp_1.

Having obtained the figure in the plane QR, let us now take for true ground plane the horizontal plane of p_1, the ground line being XY.

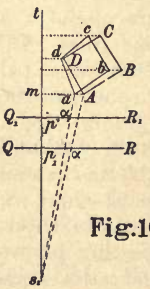

Fig. 105

Let $ABCD$, (Fig. 105), be the figure in its plane, s_1p_1 the projection of the distance line, QR the trace of the picture plane and s_1 the foot of the station. The projection of A on the true ground plane is at the same distance from the ground line as A is from QR, but the distance of this projection from s_1t is equal to mA multiplied by the cosine of the inclination ω of the plane QR, for let a', (Fig. 104), be the vertical projection of A; the right-angled triangle $p_1a'n$ gives:

$$p_1n = p_1a' \cos. \omega.$$

Therefore, if Am (Fig. 105), be drawn parallel to QR, am taken equal to Am cos. ω and the same operation repeated for B, C, and D, the resulting figure $abcd$ is the ground plan of $ABCD$.

The ground plan may be obtained in another way. For join s_1A: the intersection a with QR is the projection on QR of the point of the perspective corresponding to A. Take s_1p' equal to s_1p_1 sec. ω and through p' draw Q_1R_1 parallel to QR: join s_1a. The similar triangles s_1p_1a, s_1mA, give:

$$\frac{s_1p_1}{p_1a} = \frac{s_1m}{mA} \qquad (1).$$

From the similar triangles $s_1p'a_1$, s_1ma, we have :

$$\frac{s_1p'}{p'a_1} = \frac{s_1m}{ma} \qquad (2)$$

Dividing (1) by (2), replacing s_1p' and ma by s_1p_1 sec. ω and mA cos. ω respectively, we find :

$$p'a_1 = p_1a.$$

This means that if the perspective be moved in Q_1R_1, the directions obtained from the perspective for the different points of the plane QR will be the directions of the horizontal projections of these points.

Therefore to construct the horizontal projection of the figure seen in perspective, find the distances of the various points of the figure from the picture plane by means of a vanishing scale (§68) made with Pp_1, Fig. 104, as height of the station and the real distance line. Then find the directions of the projections, using QR as ground line and a distance line increased in the proportion $\frac{1}{\cos. \omega}$. The figure constructed with the above distances and directions is the horizontal projection of the figure in the plane QR.

80. Change of ground plane and distance line.—Let A, (Fig. 106), be a point of a figure in a plane perpendicular to the picture plane and a its perspective. Take the plane of the figure as ground plane and let s_1p_1 be the trace of the assumed principal plane. Revolve this principal plane around its trace on the ground plane : the station comes in S, b and b' being the projections of a and a' the projection of A on the assumed principal plane. Move the perspective to $b_1b'_1$ so that $s_1p' = \frac{s_1p_1}{\cos. \omega}$, ω being the angle of the assumed and true ground planes ; it has been shown that s_1b_1 is the direction of the projection a of A on the true ground plane revolved around s_1p_1 on the assumed ground plane. The visual ray, however, does not pierce the ground plane in a, its projection on the principal plane having been changed from Sb' to Sb'_1 by the displacement of the perspective. But join Sp' and take as new ground plane the plane passing through c : $s'a'_1$ is the trace of the assumed principal plane on this new ground plane and a'_1 the projection of the trace of the visual ray on the last plane. Consequently this trace is at the intersection of s_1a with the perpendicular drawn from a'_1 to s_1a'.

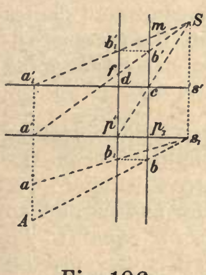

Fig. 106

Similar triangles give the following proportions :

$$\frac{p'a'}{b'b'_1}=\frac{p'f}{fb'_1}=\frac{cb'}{b'm}=\frac{db'_1}{b'm}=\frac{da'_1}{b'b'_1}$$

or

$$\frac{p'a'}{b'b'_1}=\frac{da'_1}{b'b'_1}$$

and

$$p'a'=da'_1$$

$p'a'$ being equal to da'_1 the figure $p'da'_1a'$ is a parallelogram and a'_1a' is perpendicular to s_1a', therefore the visual ray will pierce the new ground plane in a.

Hence, if the perspective be moved from p_1 to p' and $s'a'_1$ taken as ground plane, the perspective viewed from the station corresponds on the new ground plane to the projection of the figure on the true ground plane: this projection can consequently be constructed by the methods of §§65, 66, 67.

Fig. 106 gives the proportion :

$$\frac{Ss'}{Ss_1}=\frac{s'c}{s_1p'}=\cos.\ \omega.$$

or

$$Ss'=Ss_1\cos.\ \omega.$$

The heights Ss', Ss_1, of the station above the various ground planes being equal to the distances of the principal point from the corresponding ground lines, the new ground and distance lines can be found as follows :

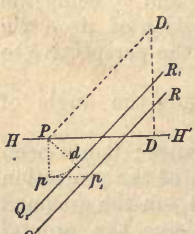

Fig. 107

Let QR, (Fig. 107), be the trace on the picture plane of the plane containing the figure. From the principal point P, let fall Pp_1 perpendicular to QR and draw Pp and p_1p perpendicular and parallel to the true horizon line HH'. Take Pd equal to Pp and draw Q_1R_1 parallel to QR: it is the ground line to be used in the construction, because

$$Pd=Pp_1\cos.\ \omega.$$

At the distance point, draw DD_1 perpendicular to HH' and draw PD_1 parallel to QR: PD_1 is the length to be used as distance line.

The height of the station Pd used for the construction is always smaller than the real height Pp_1 above the plane of the figure, therefore the precision of the construction is less than if the figure had been in a horizontal plane.

81. From the perspective of a figure in any given plane, to construct the horizontal projection of the figure.—The method of squares can be again employed in this case. Let QOR, Fig. 108, be the traces of the plane of the figure on the ground and picture planes, and $abcd$ one of the squares of the ground plan. The projecting plane of ad intersects the traces of QOR in Q and L; the vertical projection of the intersection of the two planes being Lq'. Through the station draw a parallel to ad, Lq': the horizontal projection is sv parallel to ad, the vertical projection is PV parallel to Lq' and the vertical trace, V, is the vanishing point of the intersection of QOR with the projecting plane of ad. The perspective of this intersection is VL: the perspective of the intersection of the projecting plane of cb is VK and all the lines required may be drawn in perspective by carrying the distance LK on the trace OR and joining the points of division to V.

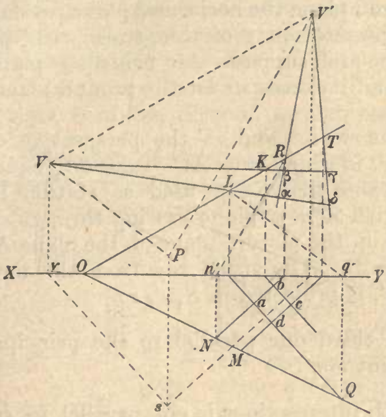

Fig. 108

The perspectives of the intersections with the plane QOR, of the projecting planes of ab and cd are obtained in a similar manner by drawing through the station a parallel to ab, $n'R$, for instance, and joining the vanishing point V' to R and T. The resulting figure $\alpha\beta\gamma\delta$ corresponds, on the perspective, to the square $abcd$ of the ground plan.

It is also possible to construct a vanishing scale (§ 68) for measuring the distances of the various points from the picture plane.

Through the station, a plane is drawn perpendicular to the vertical trace of the given plane: the intersections of the latter with the picture and perpendicular planes and the station point are placed in their actual positions and the vanishing scale is constructed by measuring equal distances from the trace of the picture plane.

82. Change of station, ground and picture planes.—The same result is arrived at by changes in the relative positions of the station, perspective and ground planes.

Let QR, (Fig. 109), be the trace, on the principal plane, of the plane containing the figure, which we will call A. Take for ground plane the horizontal plane passing through the intersection p of this trace with the principal line and suppose the principal plane revolved around its trace sp on the ground plane.

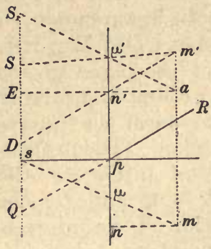

Fig. 109

Let S be the station and $\mu\mu'$ the perspective of the point mm', in the plane A. Take SS_1 equal to Qs and suppose that S_1 be used as station in connection with a new plane passing through sp and the trace on the picture plane of the plane A. Call this plane B. The visual ray from the new station to μ,μ', is projected in $S_1\mu'$, $s\mu$.

Cut the planes A and B by a third one parallel to the principal plane and passing through the point mm'.

The horizontal projection of both intersections is mn, parallel to sp. The projection on the principal plane of the intersection with plane A is $m'n'$ parallel to QR and the intersection with plane B is projected in $n'a$ parallel to sp.

Join $S\mu'$ and produce it to m'; produce $S_1\mu'$ to its intersection with $n'a$, $m'n'$ and $n'a$ to their intersection with S_1Q. Join $m'a$. Similar triangles give:

$$\frac{n'\mu'}{SD} = \frac{m'\mu'}{m'S} \qquad (1)$$

$$\frac{n'\mu'}{S_1E} = \frac{a\mu'}{aS_1} \qquad (2)$$

But $SD = SE + ED = SE + sQ = SE + S_1S = S_1E$, hence the first terms of (1) and (2) are identical and we have:

$$\frac{m'\mu'}{m'S} = \frac{a\mu'}{aS_1}$$

which is transformed into:

$$\frac{m'\mu'}{\mu'S} = \frac{a\mu'}{\mu'S_1}$$

The triangles $SS_1\mu'$ and $am'\mu'$ having one angle equal and the two sides about it proportional, are similar, and $m'a$ is parallel to SS_1. Consequently a is on the perpendicular $m'm$ to sp.

The line sm, S_1a, is the visual ray from the new station through the point $\mu\mu'$ of the perspective: mn, an', is a line of the plane B. These two lines intersect since the intersections m and a of their projections are on the same perpendicular to the ground line, and the point of intersection is the trace of the visual ray on the plane B since the line mn, an' is in that plane. The same point is also the trace on the plane B of the vertical through mm'.

Therefore, if verticals are drawn from all the points of the figure in plane A, their traces on plane B form a new figure which corresponds to the perspective viewed from S_1.

The problem is thus reduced to construct from its perspective the horizontal projection of a figure contained in a plane perpendicular to the picture plane, which is done by a change of ground and picture planes (§ 79). The process now involves changes of station, ground plane, picture plane and trace of principal plane as follows:

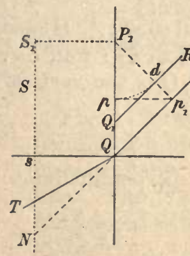

Fig. 110

Let QP_1, (Fig 110), be the principal line. Revolve the principal plane on the picture plane around QP_1, the front part of the principal plane being turned to the left: the station comes in S, and NS_1 is the vertical of the station. Let TQR be the plane containing the figure seen in perspective. Draw Qs perpendicular to QP_1 and take SS_1 equal to sT. Draw S_1P_1 parallel to sQ. The point P_1 is to be used as principal point of the perspective.

Draw P_1p_1 perpendicular to QR, pp_1 parallel to sQ and take P_1d equal to P_1p. Through d draw Q_1R_1 parallel to QR; it is the assumed ground line.

Produce QR to N: QN is the length to be assumed as distance line.

On the constructed figure, the perpendicular to the picture plane at p_1 will appear as trace of the principal plane on the ground plane.

The traces of the plane containing the figure are found as in § 75.

83. Reflected images.—The case of horizontal reflecting surfaces is the only one that will be considered.

When a perspective contains the direct and reflected images of the same point, the point can be located in space, provided the altitude of the station above the reflecting surface be known.

Take for ground plane the reflecting surface and revolve the principal plane on it, around its trace. Let a, a', Fig. 111, be the point in space, a, a' its perspective and a a'_1 the perspective of its reflected image. The horizontal projection is the same for both images, because the reflecting surface being horizontal, the direct and reflected visual rays are in the same vertical plane having for trace sa.

Let sa, SOa', be the reflected visual ray: according to the laws of reflection, the direction of SO is the same as if a' were placed at a distance equal to ca' below the reflecting surface and on the same vertical.

Produce $a'O$ to S_1 : cb being equal to ca', sS is equal to sS_1. Hence to find the position in space of a, a', take sS_1 equal to sS : join Sa', Sa'_1 and S_1O : the point of intersection of Sa' and S_1O is the vertical projection of the point of space.

Join sa and produce to the intersection with $a'a$, perpendicular to the ground line; aa' is the required point.

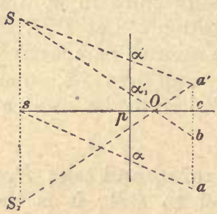

Fig. 111

The construction gives not only the position a of the point on the ground plane, but also its height ca'.

The middle of the vertical between the direct and reflected images corresponds to a, the horizontal projection of the point on the ground plane. This shows that when the shore of a lake, for instance, is indistinct on a perspective, it would be incorrect to take for shore line the middle line between objects and their images in the lake, because this would give for the distance of the shore that of the objects themselves.

84. Shadows.—The subject of shadows is an important branch of perspective, but only those cast by the sun need be considered here.

Let α and β (Fig. 112) be the perspectives of two points A and B, m and n their shadows. The line joining A to its shadow is the direction of the sun, and so is the line joining B to its own shadow; therefore these lines are parallels and their vanishing point is V, at the intersection of $m\alpha$ and $n\beta$.

A line drawn from the station to the sun is parallel to the first two lines, because it is also the direction of the sun; therefore V is its trace on the picture plane or the perspective of the sun.

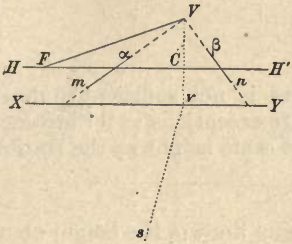

Fig. 112

From V draw Vv perpendicular to XY; sv is, on the ground plane, the direction of the sun. On the horizon line take CF equal to sv and join FV: FVC represents, revolved on the picture plane around VC, the triangle having its vertex at the station and VC as opposite side. Therefore VFC is the altitude of the sun. Having the sun's altitude, the azimuth of the line sv of the ground plan can be calculated, provided the latitude and approximate time are known.

Fig. 112 represents the sun in front of the observer. When it is behind, the line between the station and the sun does not pierce the picture plane; it has to be produced to intersect it below the horizon line. The trace of this line on the picture plane, V, Fig. 113, is still considered as the perspective of the sun; it is obtained in the same manner as when the sun is in front and all demonstrations apply to one case as well as to the other.

The calculation of the azimuth can be made by the method given in § 37 for the solution of spherical triangles.

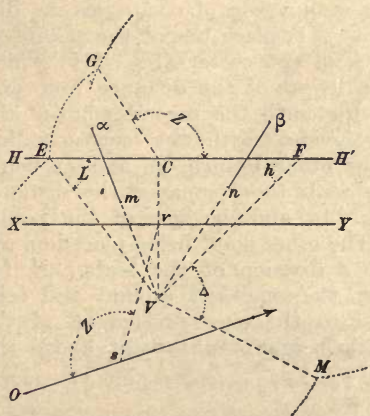

Fig. 113

Find the altitude CFV of the sun by the construction given above, make EVC equal to the colatitude of the place and FVM to the polar distance of the sun. Take VM equal to VE and from C and F as centres with CE and FM respectively as radii, describe arcs of circle. Join their point of intersection, G, to C, and GCF is the azimuth of the sun.

When the perspective has been taken in the morning, plot the angle Z on the left of sv in vsO, and the line Os is the north and south line of the ground plan.

In the afternoon, the angle Z should be plotted on the right of sv. The rules are reversed when the perspective of the sun is above the horizon line.

85. Heights.—As a rule, one perspective is not sufficient to determine the height of a point, although there are exceptions, as for instance, points on the horizon line which are at the same height as the station.

The horizontal projection of the point being known, the height above the ground plane is measured with a scale in the same manner as a vertical is divided into equal parts (§ 61).

For instance, a and a, Fig. 114, being the perspective and horizontal projection of a point, and s the foot of the station, draw aF parallel to XY.

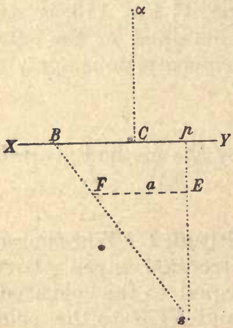

Fig. 114

From the trace p of the principal line, take pB equal to the distance of a from XY. Join sB, and FE is the height of the point above a.

This height being a fourth proportional to three known lines, can be found with an ordinary sector. Take with a pair of compasses the distance from a to XY, place one of the points on the division p of the sector (Fig. 115) which expresses the length of the distance line, and open the sector until the second point of the compasses coincides with the corresponding division of the other branch, sp and sB being equal. Now take with the compasses the distance from a to XY (Fig. 114) and place one of the points in p (Fig. 115). The other point being placed on sp, will coincide with a division of the scale, E for instance; then turn the compasses around and take the distance from E to the same division F of the other scale; EF is the height of the point above the ground plane.

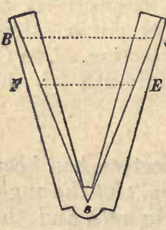

Fig. 115

86. Relations between two perspectives of the same object.—Prof. G. Hauck* has shown some useful properties of two perspectives of the same object taken from different stations. In Fig. 116, S and S' are the two stations, σ is the perspective of S' on the picture plane of S, and σ' is the perspective of S on the picture plane of S'. Prof. Hauck calls σ and σ' the *kern points*. For facility of reference, we will call any plane containing the two *kern* points, a *kern* plane.

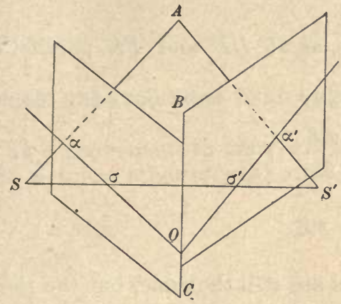

Fig. 116

Let SAS' be such a plane passing through the point A of the object. The intersection BC of the two picture planes pierces the *kern* plane in O; $O\sigma$ and $O\sigma'$ are the traces of the last plane on the first ones, $O\sigma$ is therefore the perspective of $O\sigma'$ and inversely $O\sigma'$ is the perspective of $O\sigma$. The five lines SA, $S'A$, SS', $O\sigma$ and $O\sigma'$ are in the *kern* plane; the intersection α of SA and $O\sigma$ is the perspective of A from S and the intersection α' of $S'A$ and $O\sigma'$ is the perspective of A from S'. Hence the rule that the lines $\alpha\sigma$ and $\alpha'\sigma'$ joining the two perspectives of the same point to the corresponding *kern* points meet the intersection BC of the picture planes at the same point O.

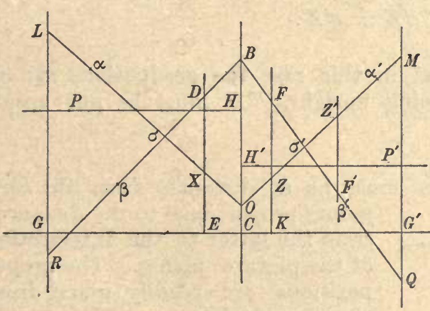

Fig. 117

Revolve the two picture planes around their intersection BC, until they coincide (Fig 117); GG' is the ground line, P, P' are the principal points. If a scale is placed on BC, the zero being on the ground line GG', it is intersected at the same division O by $\alpha\sigma$ and $\alpha'\sigma'$, and the space intercepted in HH' between the two horizon lines is equal to the difference of altitude of the stations. So when the perspectives are separated, scales placed on the line BC on the two perspectives with their zeros on the ground lines, are intersected at the same division by $\alpha\sigma$ and $\alpha'\sigma'$. The same

* Theorie der trilinearen Verwandschaft ebener Systeme. Journal für reine und angewandte Mathematik. 95 Band, 1883.

relation holds good for the perspectives β and β' of any other point; $\beta\sigma$ and $\beta'\sigma'$ meet the same division B of the scales.

The scales may be placed elsewhere, as at DE and FK parallel to BC, provided

$$\frac{\sigma O}{\sigma X} = \frac{\sigma' O}{\sigma' Z}$$

Similar triangles show that

$$DX = FZ$$

The spaces intercepted on the scales are still the same, but the zeros are no longer on the ground line unless the stations are at the same altitude.

The scales can be placed on the opposite sides of the *kern* points at LR and MQ. The spaces intercepted are equal provided

$$\frac{\sigma O}{\sigma L} = \frac{\sigma' O}{\sigma' M}$$

Instead of setting the scales in DX and FZ, the second one may be placed at $F''Z'$. The spaces intercepted, DX and $F''Z'$, are equal when

$$\sigma' Z = \sigma' Z'.$$

But, it must be observed that in this case the graduations run in opposite directions; DX being upright, $F''Z'$ must be put upside down.

As a general rule, the scales must be at distances from the *kern* points proportional to the distances from the latter to the intersection of the picture planes. The proper positions are readily found from the ground plan.

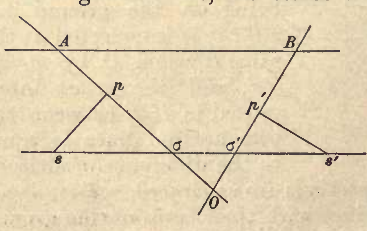

Fig. 118

Let $A\sigma$, $B\sigma'$, (Fig. 118), be the traces of the picture planes, p, p' the traces of the principal lines and s, s' the stations. Draw AB parallel to ss'. A scale parallel to, and at the distance Ap from the principal line of the first perspective

will correspond to a scale parallel to, and at the distance Bp' from the principal line of the second perspective, because

$$\frac{\sigma O}{\sigma A} = \frac{\sigma' O}{\sigma' B}.$$

which is the condition to be fulfilled.

It is convenient to draw AB so that both points shall lie outside of the pictures, otherwise the scales would hide portions of the views.

CHAPTER III.

Perspective Instruments.

87. Simplest form of perspective instrument.—Many instruments have been devised for producing perspectives, either by mechanical or optical means.

One of the simplest forms is probably the wire grating represented in Fig. 119. Wires are stretched on a frame so as to divide it into small squares. The frame is placed in front of the object or view to be reproduced, and the draughtsman looks through an eye-hole in a fixed position. Dividing his paper into squares in the same manner as the frame, he is able to reproduce the outlines of the subject by drawing his lines through the squares of the paper corresponding to those of the frame. The distance from the frame to the eye-hole is the distance line of the perspective when the squares of the paper are equal to those of the frame.

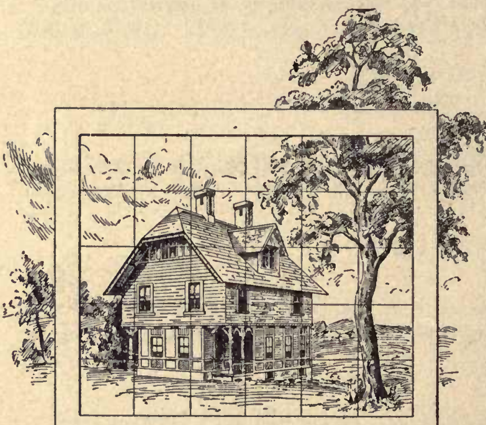

Fig. 119

88. Diagraph.—While for artistic purposes the grating is quite sufficient, there is some uncertainty in drawing the figures of the corresponding squares. To obviate this defect, it has been proposed to follow the outlines of the subject with a pointer moved by the hand, as in Fig. 120.

A drawing board, on which is stretched a piece of paper, is placed in an upright position in front of the subject of the perspective. It is provided with a rod, supported at both ends by cords attached to a counterpoise at the back of the board. The rod may be moved up or down and to right or left, but, owing to the mode of suspension, it is

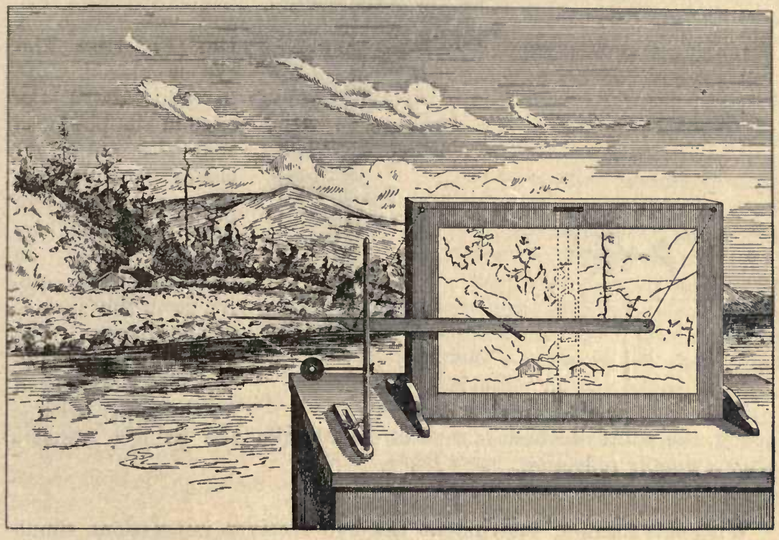

Fig. 120

always parallel to the same direction. At the middle it carries a pencil resting on the paper and at the end it has a pointer corresponding to the eye-hole of the instrument. The draughtsman takes the pencil with his hand and placing his eye at the eye-hole, he follows with the pointer the outlines of the subject by moving the rod in the proper direction. The pencil reproducing exactly the motion of the pointer, describes the perspective on the drawing board.

The plane in which the pointer moves is the picture plane, the eye-hole is the station and its shortest distance from the pointer is the distance line.

The upright position of the drawing board is inconvenient ; in a modification of the same instrument called the "Diagraph," the paper is placed horizontally and the motion of the pointer is transmitted to the pencil by a cord and pulley.

None of these instruments have come into general use.

89. Camera lucida.—The camera lucida is the invention of Wollaston. It consists essentially of a four sided prism having a right angle, two angles of 67° 30′ and one angle of 135° (Fig. 121). The eye is placed close to and above the edge of the prism, so that the pupil receives at the same time the rays of light emitted by objects placed in front and those coming from the surface of the paper. The outlines of the subject may be followed on the paper with a pencil, the point of which is seen directly while the subject appears after a double reflection.

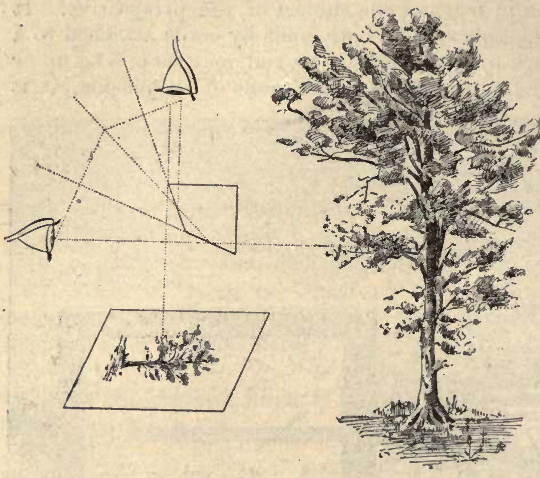

Fig. 121

With this form of instrument the eye receives impressions simultaneously from objects at different distances. The pencil and paper are quite close and the object is generally far away. The eye cannot accommodate itself to both distances; one of the images is always more or less confused, and the work is very trying.

In looking at a point of the perspective through the diagraph or the camera lucida, it will be noticed that the corresponding position of the pencil is liable to slight variations. This is the effect of parallax, and will be understood better by considering a perspective drawn on a pane of glass, B C (Fig. 122), placed in front of the subject. By looking through the eye-hole D, the distant point A will be seen in the directions Ea or Fb, according to the

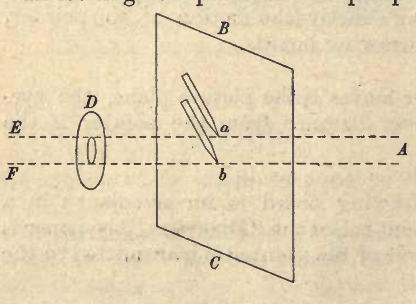

Fig. 122

position of the eye. The pencil may thus be placed either at a or b and still coincide with A. The parallax or displacement of the pencil is equal to the diameter of the eye-hole, and might be eliminated by making the hole very small, but practically it cannot be made smaller than the pupil, because it would cut off part of the light entering the eye and the images would be too dark. The parallax can be eliminated otherwise.

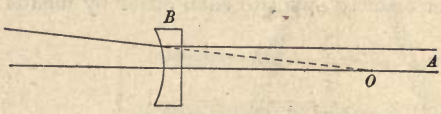

Fig. 123

Substitute for the eye-hole a plano-concave lens B (Fig. 123); the parallel rays from A will, after passing through B, diverge from O, the focus of the lens, and the appearance to the eye is as if A had been brought to coincide with O. By placing such a lens between the subject and the prism of the camera lucida, close to the latter, and selecting a lens of a focal length equal to the distance between the prism and the paper, the virtual image of the subject is brought into the plane of the paper, the parallax disappears, and the eye, accommodating itself to this one distance, is relieved from strain. Wollaston, to whom this improvement is due, explained that the same effect was obtained by making the upper face of the prism concave and dispensing with the plano-concave lens.

It was with the camera lucida, in 1849 and 1850, that the first perspective surveys were made. The improved instrument which Col. Laussedat devised for surveying is the hemi-periscopic camera lucida (1). He takes 15 centimetres for the radius of the spherical concavity in the upper face of the prism, and places the centre of the sphere on a perpendicular to the upper face of the prism passing through its inner edge (Fig. 124). The nodal point is thus located very nearly on the inner edge of the prism, which is also its axis of rotation. Unless these two conditions are fulfilled a motion of the prism round the axis causes a displacement of the image on the paper. The focal length is about 30 centimetres, which is the average distance of distinct vision, and for which the parallax of the instrument is entirely eliminated. It may still be used within certain limits at shorter or greater distances from the paper without any great amount of parallax, but it must be noted that

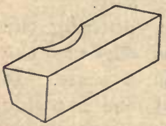
Fig. 124

(1) Col. Laussedat's investigations were first published in the "Memorial de l'officier du genie" for 1854. Very full instructions for the use of the camera lucida in surveying are given in the "Annales du Conservatoire des Arts et Metiers for 1891.

it would not be available for copying or enlarging drawings, although it might answer for reducing. In copying a drawing full size, the parallax is the same as when looking at a distant point through a prism with a plane upper face. For work of this kind, the mounting of the prism should have a second eye hole to the right or left of the concave portion. The parallax may then be eliminated and the rays of light brought to the same divergence by means of auxiliary lenses.

Laussedat's camera lucida is fixed to the drawing board by two arms with clamps, which are set exactly opposite each other by means

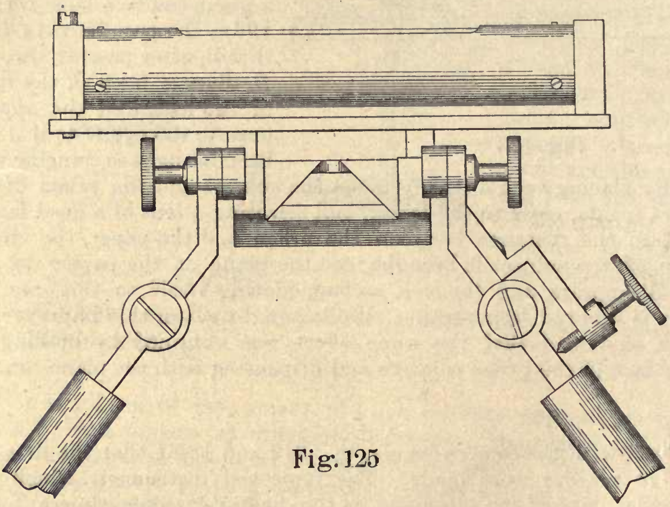

Fig. 125

of the scales drawn on the edges of the board (Fig. 126). The adjustments are made in the following order :—

1. The drawing board is levelled in the same way as a plane table.

2. The upper face of the prism is levelled by means of the level shown above (Fig. 125) and the slow motion screw at the side.

3. The principal point is found with a plumb line touching the edge of the prism (Fig. 126). The distance from this edge to the principal point is the distance line : it is measured with a scale. It can be made longer or shorter by changing the length of the two side arms of the instrument.

4. Horizon and principal lines—Suspend a plumb line at some distance in front : turn the drawing board around its vertical axis till the image of the plumb line is seen passing through the principal point. This image is the principal line : it is traced with the pencil. A perpendicular through the principal point is the horizon line. Other constructions for these lines will be found in Col. Laussedat's memoir.

The angles of the prism must be very accurate: otherwise, their errors must be calculated and taken into account.

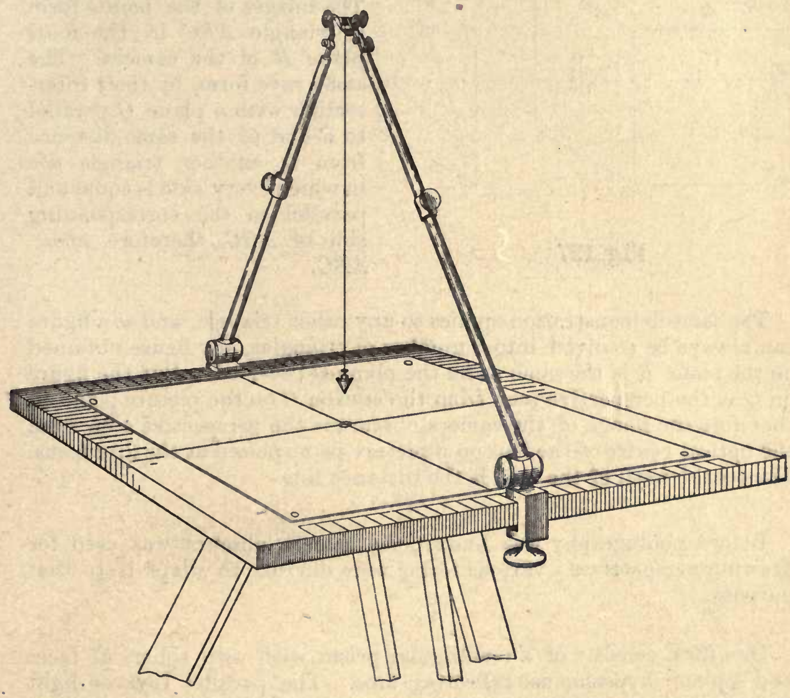

Fig. 126

Neutral tint glasses placed between the eye and the paper, or the eye and the subject, serve to equalize the brightness of the images.

This form of camera lucida is a perfect surveying instrument.

90. Camera obscura.—A camera obscura, in its simplest form, is a box hermetically closed to extraneous light, except that coming through a lens placed on one of the sides. The opposite side of the box being in the focal plane of the lens, an image is formed on it of the distant objects situated in front of the lens.

Disregarding the errors introduced by lenses, the image of the camera obscura is a true perspective, for it is the same as would be drawn on a picture plane placed in front of the lens at a distance equal to the focal length.

Let O, (Fig. 127) be the optical centre of the lens and OA, OB, OC, three rays of light coming from three distant points of space. The images of the points form a triangle ABC in the focal plane R of the camera. The same rays form, by their intersection with a plane Q parallel to R and at the same distance from O, another triangle abc in which every side is equal and parallel to the corresponding side of ABC, therefore, $abc = ABC$.

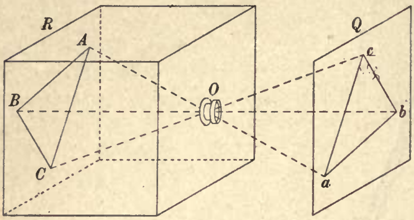

Fig. 127

The same demonstration applies to any other triangle, and as a figure can always be resolved into a number of triangles, any figure obtained on the plane R is the same as on the plane Q reversed. But the figure on Q is the perspective seen from the station O on the picture plane Q, therefore the image of the camera obscura is the perspective seen from the optical centre of the lens on a picture plane placed at the first focus. The focal length of the lens is the distance line.

Before photography was known, the camera obscura was used for drawing perspectives; various forms were devised to adapt it to that purpose.

One form consists of a rectangular prism with two spherical faces and a plane hypothenuse reflecting face. The parallel rays of light emitted by the subject are brought to a focus by the two spherical faces while they are reflected at right angles by the hypothenuse face.

The prism is placed on the top of a tripod, which supports a drawing board at the proper height to receive the image formed at the focus of the object glass. The tripod is covered with a black cloth to shut off extraneous light, so as to enable the draughtsman to see the image projected on the paper and to follow it with a pencil.

The point of the pencil being between the lens and the paper, casts a shadow just at the point where the image is wanted. The instrument shown in Fig. 128 is not open to the same objection and requires neither tripod, drawing board, nor even a black cloth. It is merely a box with a lens in front and a mirror in PQ inclined at 45° to the axis of the box.

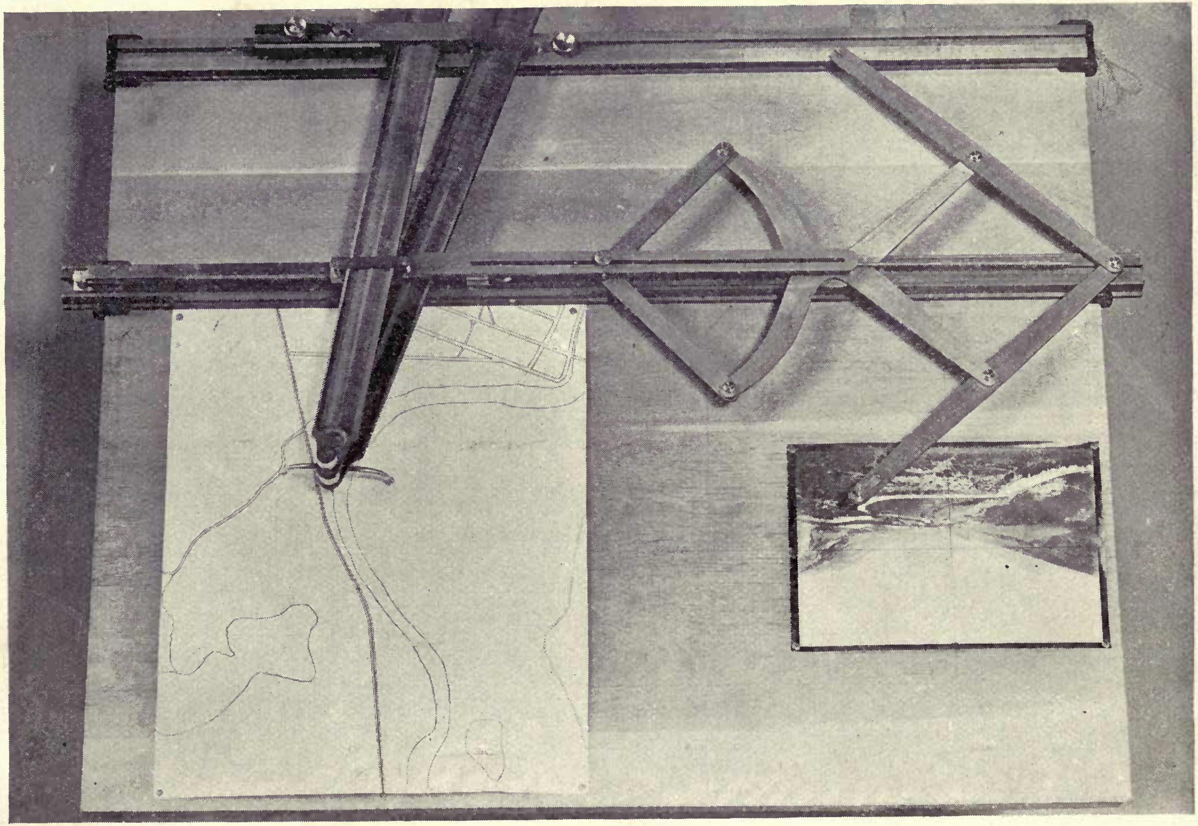

Fig. 129—PERSPECTOGRAPH.

The image formed by the lens is reflected by the mirror on a ground glass placed in AB; being inverted a second time by the reflection, it now appears upright. The lid which covers the ground glass, when not in use, is open at an angle of about 45°, and cuts off sufficient light for the image to be visible. Under these conditions, the image is not bright enough to work on paper and has to be traced on the ground glass, but with a black cloth covering the box and the head of the draughtsman, it is possible to work through thin paper.

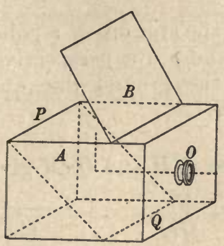

Fig. 128

91. Perspectograph.—The perspectograph is the invention of Hermann Ritter, a German architect (1), its object being to draw a perspective from the plans of the subject and not from the subject itself. The lines of the plans are followed with a tracer, and the perspective is drawn by a pencil carried by another part of the instrument.

As constructed by Chs. Schroder & Cie (Frankfort-on-the-Main), it is a large instrument made partly of wood and partly of metal; well adapted for drawing perspectives of buildings from an architect's plans, but useless for drawing a topographical plan from a perspective. It is represented in Fig. 129.

For surveying purposes, the instrument should be of small size, made entirely of metal, and all the parts should be fitted with precision. It should work easily and the amount of dead motion should be as small as possible.

Its theory is, with slight modifications, applicable to any other perspective instrument and for that reason is given here at length, but the use of the perspectograph, in its present form, cannot be recommended for photographic surveys.

However perfect an instrument may be it always introduces in the final result some errors of its own, due to dead motion, to imperfections in the adjustments, and to the slight errors unavoidable in the determination of the constants. Whenever the precision of the survey requires it, a geometrical construction should be employed, and the use of perspective instruments should be restricted to reconnaissance or rough surveys, in which rapidity is more important than perfect accuracy.

(1) Perspectograph, von Hermann Ritter, Architekt, Frankfurt a. M. Druck von J. Maubach & Co.

The principle of the apparatus is as follows:—

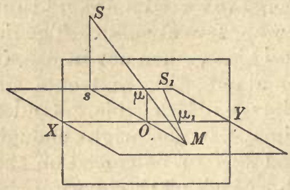

Fig. 130

Let S (Fig. 130) be the station, M a point of the ground plane and μ its perspective. Take sS_1 parallel to the ground line and equal to sS; join S_1M.

The similar triangles sSM and $O\mu M$ give

$$\frac{sS}{O\mu} = \frac{Ms}{MO}$$

From the triangles sS_1M and $O\mu_1M$, also similar, we have:

$$\frac{sS_1}{O\mu_1} = \frac{Ms}{MO}$$

Hence,

$$\frac{sS}{O\mu} = \frac{sS_1}{O\mu_1}$$

But, by construction, sS is equal to sS_1, therefore

$$O\mu = O\mu_1$$

This relation furnishes a new method for constructing a perspective. Take on the ground plan (Fig. 131) sS_1 parallel to the ground line and equal to the height of the station. To find the perspective of a point M of the plan, join sM and S_1M, which intersect the ground line in O and μ_1. On another part of the paper, draw the ground line of the perspective $X'Y'$ and take on it a point O' to represent the point O of the ground plan. At O' erect $O'\mu$ perpendicular to $X'Y'$ and equal to $O\mu_1$; μ is the perspective of M. Owing to the position of the figure, the perspective appears upside down.

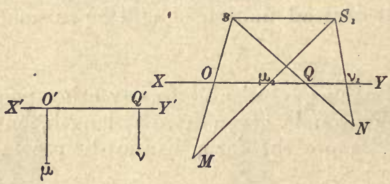

Fig. 131

The perspective of another point N of the ground plan is obtained in a similar manner by taking $O'Q'$ equal to OQ and $Q'\nu$ equal to $Q\nu_1$.

This is done mechanically by the perspectograph: sM and S_1M (Fig. 132) are two wooden arms joined in M and carrying the tracer. They slide through four adjustable pieces—s, S_1, O and μ_1: s and S_1

can be adapted to any part of a rule RT, s is fixed at the point of the ground plan representing the foot of the station, and the rule or slide RT is firmly clamped to the drawing board parallel to the ground line. The second piece S_1 is placed at a distance from s equal to the height of the station and fixed in that position.

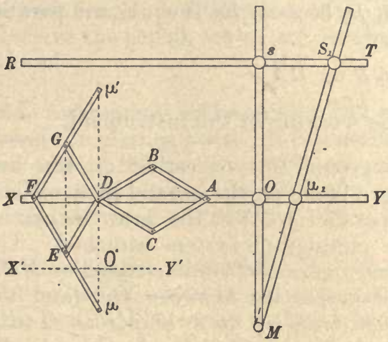

Fig. 132

The third piece O is attached to a rod which moves in the groove of a slide XY, and carries a pantograph system, with axis at D fixed to the rod, so that the distance from O to D is invariable while the instrument is in use. When the arm sM is moved, s being a fixed point, O follows the motion of the arm, and carries with it, along the groove XY, the movable rod and the pantograph system.

The fourth piece μ_1 is connected with the joint A of the pantograph system, so that the distance $\mu_1 A$ is invariable during the operation; it is also bound to slide on the movable rod.

The pantograph system is composed of four straight arms (AB, AC, $F\mu$ and $F\mu'$) and two arms (CDE and BDG) bent at right angles in D. They are joined in A, B, C, D, E, F and G, the sides of the parallelograms $ABDC$ and $DGFE$ being all equal. The arms $F\mu$ and $F\mu'$ are double the length of one side of the parallelograms, and the pencil which is to describe the perspective may be placed either in μ or μ'.

The sum of the four angles at D is equal to four right angles; two of these angles, CDE and BDG, being right angles, the sum of the two remaining ones must be equal to two right angles; that is

$$CDB + EDG = 180^\circ.$$

But in a parallelogram the sum of two adjacent angles is equal to two right angles; that is,

$$CDB + DCA = 180^\circ$$

hence,

$$EDG = DCA.$$

Therefore the two parallelograms are equiangular and their sides being equal, the parallelograms are equal, but not placed in the same direction. The diagonal DA of one is equal to the diagonal GE of the other, and BC is equal to DF.

The line $\mu\mu'$ is parallel to GE because $F\mu$ is equal to $F\mu'$; it is therefore perpendicular to XY since the diagonals of a rhombus intersect at right angles, and it passes through D, because $E\mu$ is equal and parallel to GD. We have also

$$D\mu = GE = DA$$

It is now easy to understand the working of the instrument.

The slide XY is placed on the ground line or rather on the line representing the trace of the picture plane on the ground plan. When the tracer in M is moved on a parallel to XY, the arm Ms carries with it the movable rod and the pantograph system attached. The distance from O to μ_1 does not vary, since the similar triangles MsS_1 and $MO\mu_1$ always give the same proportion between $O\mu_1$ and the constant length sS_1. The distance from μ_1 to A and from O to D being invariable, and $O\mu_1$ being constant, AD and consequently $D\mu$ do not change, and the pencil in μ describes a parallel to XY: it is the perspective of a line of the ground plane, parallel to the picture plane.

When the tracer is moved away from XY, in the direction of Ms, the points O and D do not change, but $O\mu_1$ is lengthened and μ_1 moves towards the right carrying with it the joint A and increasing the diagonal DA to the same extent as $O\mu_1$; $D\mu$ being equal to DA, is also lengthened and μ moves down, precisely the same distance as μ_1 moved to the right.

The construction thus effected mechanically is that of Fig. 131. The ground line of the perspective, $X'\ Y'$, is the line which would be described by the pencil in μ, if the tracer M could be brought to the centre of the groove and moved along XY: O and μ_1 would then coincide.

Drawing the tracer away from XY, but in the direction sM, μ_1 separates from O, and μ moves down by the same quantity from its former position O' on the ground line, $O'\mu$ being perpendicular to $X'Y'$ and

$$O'\mu = O\mu_1$$

Now if M be placed on any other point of the ground plan, the perpendicular $D\mu$ to $X'Y'$ will be carried away the same distance as the point O, and μ will be at a distance from $X'Y'$ equal to the new value of $O\mu_1$.

The perspective is upside down, the draughtsman having to place himself near M to guide the tracer.

The end μ' of the arm $F\mu'$ describes the symmetrical figure of the perspective, or the image which would be seen in a mirror, but were the fixed point S_1 placed on the left of s, μ' would describe the true perspective, the direction of the motions of μ and μ' being reversed. The ends μ and μ' of the arms of the pantograph system are both fitted to receive the pencil, which can be changed from one to the other as required.

The instrument, set as in Fig. 132, can only work on points or figures beyond the picture plane : it is possible to place the slide XY on the other side of RT, so as to work on points between the picture plane and the station, but the obliquity of the arms prevents them from sliding freely and the working of the instrument is unsatisfactory.

It may happen that with a high station and points at the extreme right or left (it would be the extreme left on the figure), the obliquity of the arm S_1M becomes too great to work. S_1 must then be changed from one side of s to the other, (from the right to the left of s on the figure) and the perspective from one side of the ground line to the other. The pencil is at the same time changed to the opposite arm of the pantograph system.

The sliding rod XY may be reversed end for end in its groove, the pantograph system coming on the opposite side of the movable arm Ms. The pencil does not require to be changed, but the arm bearing it, μ' for instance, instead of being between RT and XY, will now be on the other side of XY.

A scale must be drawn on RT, the zero corresponding to the pointer carried by the sliding piece s : the graduation extends to right and left, and the pointer of the sliding piece S_1 is set opposite the division corresponding to the height of the station above the ground plane.

The distance between RT and XY is equal to the distance line of the perspective.

In the case of figures in planes which are not perpendicular to the principal plane, it has been shown that the solution of problems involves changes in the distance line : the edges of the drawing board must, therefore, carry graduations to permit of XY being moved any given quantity, keeping it parallel to RT.

The different pieces of the instrument are adjustable, and must first be placed in proper position for the work in hand. This done, the

slide RT is firmly clamped to the drawing board and XY placed parallel to RT. All that is necessary now, is to draw the scales and determine the position of the various lines and points on which rests the construction of the perspective. Hitherto, it has been assumed that the points s, S_1, O, μ, etc., were mathematical points and that their distances could be measured directly, but, there is nothing on the instrument to define their exact position and no such measure could be taken with precision, consequently these quantities, which are the constants of the instrument, have to be determined indirectly.

92. To DRAW THE TRACE OF THE PRINCIPAL PLANE ON THE DRAWING BOARD.—It is assumed that the grooves in the slides RT and XY are parallel to their edges, and that the sliding motion in these grooves is also parallel to the same direction. This assumption is practically correct.

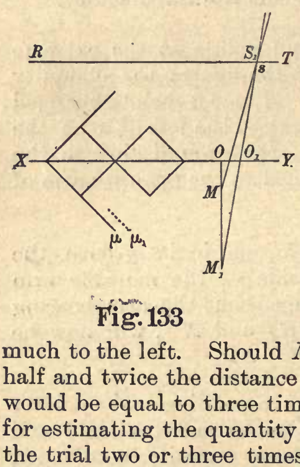

Fig. 133

Place S_1 above s (Fig. 133) and draw a perpendicular MM_1 to RT, which, if produced, would pass as nearly as possible through s. Should this line pass exactly through s, it would be the trace of the principal plane on the ground plane, and were the tracer M moved along the line from M to M_1, the point O of the slide XY would not move. To ascertain whether O has moved or not, mark the position of μ when the tracer is in M, then place the tracer in M_1 and if μ has moved to the right, in μ_1, this shows that MM_1 is too much to the left. Should MO and M_1O be equal respectively to one-half and twice the distance line, the error in the position of MM_1 would be equal to three times the displacement of μ, but it is sufficient for estimating the quantity by which MM_1 has to be shifted to repeat the trial two or three times. The motion of O is indicated by the displacement of μ to the right or to the left: a motion of μ perpendicularly to XY indicates merely that S_1 is not precisely over s.

93. To FIND THE DISTANCE FROM THE STATION TO A FRONT LINE OF THE GROUND PLANE.—The trace of the principal plane on the drawing board being now determined, the ground plan could be placed in its proper position on the board were it possible to measure exactly the distance from the point s of the instrument to a front line AB, (Fig. 134), of the drawing board. This determination is made as follows:—Draw a second line CD parallel to AB; place the two long arms one above the other and bring the tracer to M; mark the position of μ. Then carry the tracer to CD, and follow the line until the pencil has

returned to the same point μ; the line NM, if produced, would pass through s. Repeat the same operation in M_1, N_1 and μ_1. Having the lengths of MM_1 and NN_1 and the distance d between AB and CD, a simple proportion gives the distance x from s to AB:

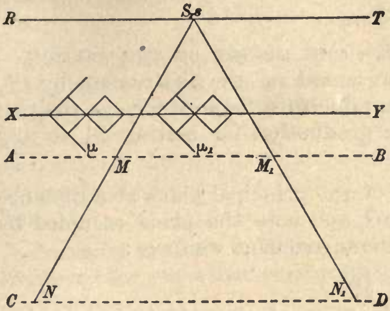

Fig. 134

$$x = d\frac{MM_1}{NN_1 - MM_1}$$

The two lines and the points NN_1 must be taken as far apart as the instrument will allow.

Having the distance of the front line AB, other front lines at fixed distances from the foot of the station are permanently marked on the drawing board.

The ground plan can now be placed on the board by putting the trace of the principal plane in coincidence with the line previously drawn on the board, and placing a front line, of which the distance is known, upon the corresponding one of the drawing board.

94. To FIND THE DISTANCE BETWEEN THE TWO SLIDES.—The distance between the two slides is equal to the distance line of the perspective; it must be indicated by a scale on the edge of the drawing board. In order to locate the zero of the graduation the precise distance has to be determined in one position of the instrument.

Draw a front line AB (Fig. 135). Put the tracer first in A and then in B, marking in each case the positions μ and μ_1 occupied by the pencil. Let d be the distance between the slides, and m the distance from s to AB; we have:

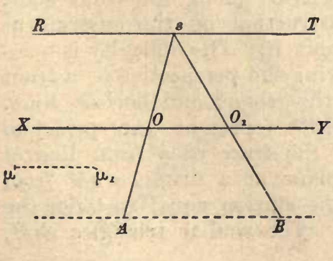

Fig. 135

$$\frac{d}{m} = \frac{OO_1}{AB}$$

AB being a front line, its perspective $\mu\mu_1$ is parallel to XY and equal to OO_1; consequently:

$$d = m\frac{\mu\mu_1}{AB}$$

The three lengths which form the second term can be measured and the value of d calculated.

95. To draw the graduation for the height of the station.—The height of the station is represented on the instrument by sS_1 (Fig. 132). It is necessary to determine this distance for one position of s and S_1 in order to draw the graduation for setting S_1 to any required height.

Place the tracer M on the trace of the principal plane at a distance Ms equal to one and a half times sO, and note the place occupied by the pencil μ. In this position of the instrument we have:

$$O\mu_1 = \tfrac{1}{3}sS_1.$$

Then place the tracer M, still on the trace of the principal plane, but at a distance Ms equal to three times sO: we have:

$$O\mu_1 = \tfrac{2}{3}sS_1.$$

The change in the value of $O\mu_1$ is thus equal to one-third of the height of the station; but, this change is represented by the displacement of the pencil μ, which can be measured with a scale. Three times this displacement is the height of the station.

The tracer, instead of being placed at the distances from the foot of the station given above, may be set at any distance which may be convenient; the fraction of the height of the station obtained will be different, but the process will be the same.

96. To draw the horizon, ground and principal lines of the perspective.—The principal line is the perspective of the trace of the principal plane on the ground plane. The latter has been marked on the board (§ 92); following it with the tracer, the pencil describes the principal line.

The ground line cannot be drawn directly, because the tracer would have to be carried along the front slide, and the construction of the instrument does not permit it. The difficulty is overcome by drawing the perspective of a front line between the ground and horizon lines.

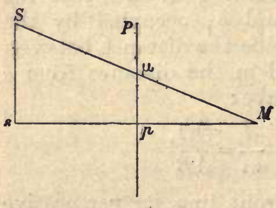

Fig. 136

Let Fig. 136 represent the principal plane and M the trace of a front line of the ground plane at a distance sM from the foot of the station equal to twice the distance line. The similar triangles SsM, μpM give

$$\mu p = Ss\frac{pM}{sM} = \tfrac{1}{2}Ss.$$

Ss is equal to the height Pp of the principal point above the ground line, therefore

$$\mu p = \tfrac{1}{2} Pp$$

Following then with the tracer the front line drawn on the board at a distance from the station equal to twice the distance line, the pencil describes a horizontal line midway between the ground and horizon lines. One-half the height of the station is now measured on each side of the line so obtained and parallels drawn to it. The line nearest to the front slide is the ground line, the other one is the horizon line.

Processes similar to those given for the perspectograph can be employed for any other perspective instrument.

97. CENTROLINEAD.—In addition to the instruments already described, others have been devised merely to facilitate the construction of perspectives. They are not, properly speaking, perspective instruments, since they do not enable the draughtsman to draw the perspective directly.

The vanishing point of a line nearly parallel to the picture plane, being at a great distance from the principal point, may fall outside the paper, in which case special constructions are necessary to draw a line which, if produced, would pass through the vanishing point.

A line vanishing at any point may be drawn with the "*centrolinead*," no matter how far it is from the principal point. The instrument consists of a straight edge (Fig. 137) with two arms whose inclination to

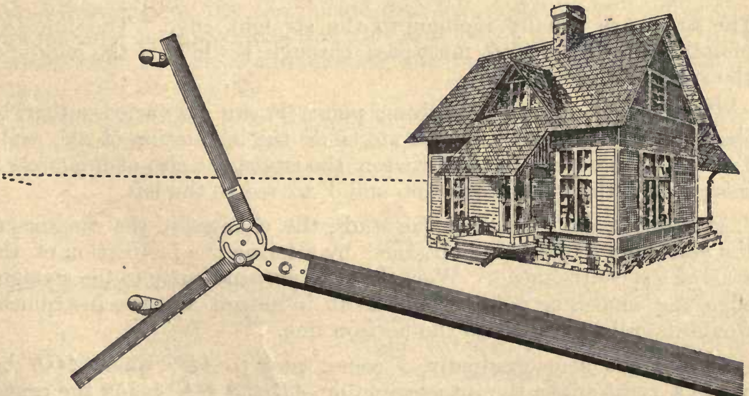

Fig. 137

the straight edge can be varied at pleasure. Two studs, six or eight inches apart, are fixed to the edge of the drawing board. The arms of

the centrolinead being placed in contact with the studs, the various directions of the straight edge intersect at a common point.

Let OC (Fig. 138) be the straight edge, OA and OB the arms, A and B the studs. Through A, O and B pass a circle; the arms being clamped, the angle O is constant and is bound to move on the circumference of the circle whenever the position of the centrolinead is changed, as from OC to $O'C'$.

Produce OC and $O'C'$ to their second intersection with the circumference: they must cut it at the same point V because the angle AOV being invariable, must always subtend the same arc AV, no matter on what point of the circumference the apex O may be. Consequently the straight edge will draw all the lines vanishing at V.

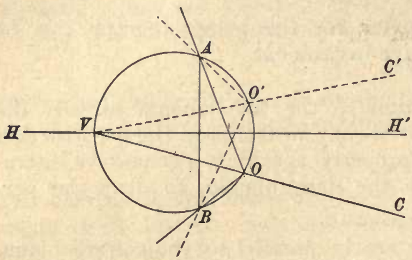

Fig. 138

In plotting surveys, the centrolinead is employed only for horizontal lines, whose vanishing point is on the horizon line. The studs A and B are placed on a perpendicular to the horizon line and at equal distances from it; it follows that the horizon line HH' is a diameter of the circle, and,

$$VA = VB$$

The arms are equally inclined to the straight edge. The line OC, bisecting the angle AOB, must pass through V which is the middle of the subtended arc BVA.

The distance of the vanishing point, V, can be varied either by changing the positions of the studs or the inclination of the arms. Increasing the distance AB between the studs, the size of the circle is increased in the same proportion and V moved to the left.

It is not usual to disturb the studs, the changes in the distance of the vanishing point being obtained by altering the inclination of the arms of the centrolinead. Were the arms perpendicular to the straight edge, the vanishing point would be at infinitum, and the instrument would describe parallels to the horizon line.

Closing the arms gradually, V comes near to AB; when AOB becomes a right angle, the intersection of AB and HH' being the centre of the circle, the distance of V from AB is one half of the latter.

Closing the arms again, V continues to move towards AB without ever reaching it.

In reality the studs are not mathematical points, but cylinders ; the direction of the straight edge is, however, the same as if the arms rested against the axes of the cylinders.

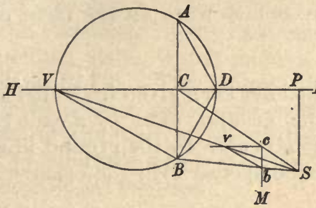

Fig. 139

The direction of the vanishing point may be given by a line of the ground plan or by a line of the perspective. In either case, the arms of the centrolinead have to be set to correspond to the vanishing point.

In the first case, revolve the picture plane on the horizon plane around the horizon line as an axis. The station comes in S (Fig. 139), SP is the distance line, A and B the two studs. Let SV be the direction of the given line on the ground plan, V is its vanishing point. Through A, B and V pass a circle ; the centrolinead should be set so that the straight edge being on DH', the arms should be on DA and DB.

Join VB ; the angle VDB, the inclination of the arm on the straight edge, is equal to the angle VBA, because they subtend equal arcs. Join CS and BS, and draw Mc and cv parallel to AB and HH'; join bv. By reason of the similarity of triangles, vb is parallel to VB and the angle

$$vbc = VBC = BDV.$$

Therefore, place the straight edge on Mb, the axis of rotation on b, and adjust the left arm of the centrolinead to coincide with bv. The other arm may be set by placing the straight edge on vb, the axis on b and adjusting the arm to coincide with bM, or better by placing the straight edge on the horizon line, the arm being already adjusted in contact with the stud B and moving the other arm until it comes in contact with the stud A.

The lines BS, CS, Mc and cv are drawn once for all, bv need not be drawn, so that the only line to be marked for setting the centrolinead is Sv, the direction of the given line on the ground plan.

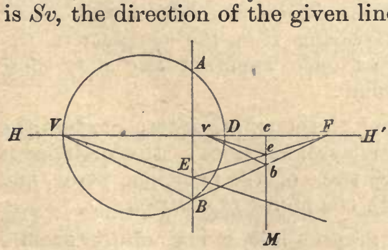

Fig. 140

When the given line belongs to the perspective, the construction must be slightly modified. VE (Fig. 140), being the given line, take any point, F, on the horizon line, join FE and FB, and draw cM parallel to AB. Through e, draw ev parallel to EV and join vb. Owing to the similarity of triangles, vb is parallel to VB and the angle vbc is equal to VBA, the inclination of the arm on the straight edge of the centrolinead.

FB and cM are drawn once for all, but FE and ve have to be marked for every given line: that is, two lines instead of one by the former construction.

Centrolineads are usually sold in pairs, one to work on the right of the principal point, and one on the left.

98. PERSPECTOMETER.—In §67, a method has been given for transferring a figure from the perspective to the ground plan by means of squares formed of lines parallel and perpendicular to the ground line. The object of the "*perspectometer*" is to dispense with the construction of the squares' perspective.

On a piece of transparent material, glass, horn or celluloid, draw two parallel lines AB, DD' (Fig. 141), and a common perpendicular Pp. Take DP, PD', pA and pB equal to the distance line, and from p, lay on AB equal distances pm, mn, pm'.... Join to P the points of division m, n, m', n'.... Part of these lines intersect AD and BD' at r, t, r', t',.... The corresponding points are joined together by lines which are parallel to AB and DD'.

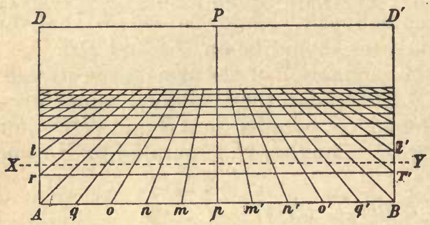

Fig. 141

The instrument is now placed on a perspective, with P on the principal point and DD' on the horizon line. The ground line falls in XY, for instance: it is divided into equal parts by the lines converging in P, and the figure of the perspectometer is the perspective of a network of squares in the ground plane having the equal parts of XY as sides. By referring to § 67, it will be seen that this is the construction given for the "method of squares."

This instrument is useful for constructing from the perspective a figure of the ground plane. By placing it on the perspective, the squares covering the figure are at once apparent and only those required are drawn on the ground plan.

The side of the squares is equal to the length intercepted on the ground line between two of the converging lines: this distance is laid on the ground plan from the trace of the principal plane, and parallels drawn to the trace through the points of division.

The front line, nearest to the ground line, is laid on the ground plan either by estimating its distance from the ground line or by constructing it. The estimation is made by noting the fraction of a square's side which represents the distance from the ground line.

Figures in planes, inclined to the horizon but perpendicular to the picture plane, are transferred to the ground plan by placing the centre of the perspectometer on the principal point and the parallel lines in the direction of the trace of the inclined plane on the picture plane. The trapezoids of the instrument are the perspectives of squares in the inclined plane, which squares are projected as rectangles on the ground plane (§ 79). The longest sides of the rectangles are perpendicular to the picture plane, and equal to the length intercepted between two converging lines of the instrument on the trace of the inclined plane. The shortest sides are the projections on the ground line of these intercepted lengths.

The rectangles are constructed on the ground plan and the transfer made from the perspective as in the preceding case.

When the plane containing the figure is inclined to the picture and ground planes, the principal point must first be displaced on the principal line, so as to project the figure on a plane perpendicular to the picture plane and having the same trace as the given plane, (§ 82) the problem is then the same as the last one.

The perspectometer can only serve for perspectives having the same distance line, such as photographs of distant objects taken with the same lens; every distance line requires a new instrument. The width Pp should be equal to the height of the horizon line above the foot of the picture; the length DD' need not be longer than the picture, the distance points being placed on the figure merely for the purpose of demonstration.

The length of the equal parts of AB should be such that the divisions of the lowest ground line employed are not too large for the degree of accuracy required. These divisions are the sides of the squares or rectangles of the ground plan and the larger their size, the less accurate will be the transfer of the figure.

The instrument can be made by drawing it on a large scale on paper, and taking a reduced negative from which a positive is made on a transparency plate. The transparency is bleached in bichloride of mercury and varnished; the lines originally black, are now white on clear glass.

99. Drawing the ground plan with the camera lucida.—The distinction between the picture and the ground planes is purely conventional; the picture plane may be taken for ground plane, and the ground plane for picture plane. If a be the perspective of the figure A of the ground plane, inversely, A is the perspective of the figure a of the picture plane. Consequently, any perspective instrument can be employed to draw the ground plan from the perspective by a change in the setting of the instrument, the distance line being now what was

formerly the height of the station, and inversely, the new height of the station being the former distance line.

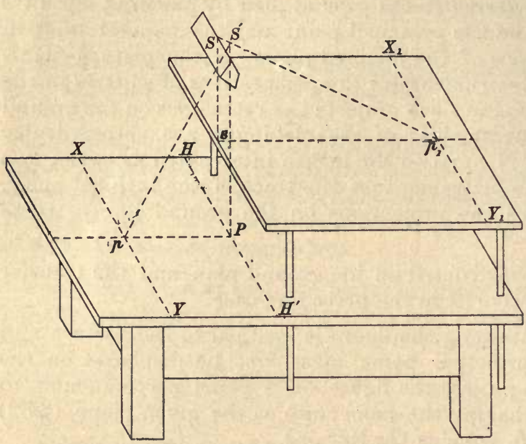

Fig. 142

With the camera lucida, the prism must be fixed permanently so that the height of the virtual station $S'P$ (Fig. 142) above the plane on which the perspective is placed, is equal to the distance line of the perspective (§89). As long as the latter does not change, the prism must remain in the same position. The perspective is placed in $HH'XY$, in such a manner that the line $S'P$ joining the virtual station to the principal point of the perspective is perpendicular to the plane of the table or drawing board.

The ground plan is on a platform sX_1Y_1, parallel to the plane of the table, and which can be moved up or down. It must be so placed that the perpendicular Ss from the real station to the platform, is equal to the height of the station; s is the foot of the station, and the ground line is somewhere in X_1Y_1.

The eye looks by reflection at the perspective and sees directly the pencil on the platform.

The determination of the constants is made by methods similar to those of the perspectograph (§ 92, 93, 94, 95, 96).

With the camera lucida, the ground plan may be drawn either within or beyond the ground line; it is a very great advantage. The parallax must be eliminated by concave or convex lenses of appropriate focal length.

100. Drawing the ground plan with the camera obscura.—In order to transfer a figure from the perspective to the ground plane, the camera obscura would have to be set as in Fig. 143, the perspective $HH'X'Y'$ being placed vertical and the ground plan horizontal in sAD. The distance from the optical centre of the lens, S, to the perspective

should be equal to the distance line, and the height of S above the plane sAD equal to the height of the station. The difficulty is that only one line would be in proper focus, the other portions of the image being more or less blurred. It might be possible, by using a lens of proper focus and a pin hole diaphragm, to obtain fair definition within a limited space $ABCD$, but it is doubtful if the process would prove practical.

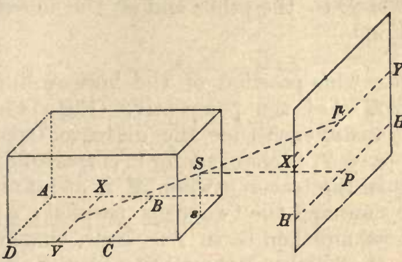

Fig. 143

101. Drawing the ground plan with the perspectograph.—In order to draw the ground plan from the perspective with the perspectograph, the distance between RT and XY (Fig. 132) must be equal to the height of the station, and sS_1 to the distance line. The principal point of the perspective must be placed in s, the horizon line under RT and the ground line under XY. The instrument thus arranged would not work with the perspectives used in surveying; it could not even be set, the obliquity of the arms being too great. It may, however, be employed to transfer a figure of the perspective to other planes than the ground plane, as, for instance, to obtain the elevation of a building from the perspective of the facade; the method is fully described in Ritter's pamphlet (1).

In other cases, and particularly for topographical purposes, the pencil must be placed in M and the tracer in μ. The perspective is placed under μ, with the ground and principal lines on the lines previously marked on the drawing board (§ 96). Taking M with the hand, the arms are moved so as to follow the lines of the perspective with the tracer μ. The operation being precisely the same as for drawing the perspective from the ground plan (§ 91), it is evident that the pencil in M will now reproduce the ground plan. The use of the instrument in this manner is at first a little difficult, owing to the point M being guided by the hand while the perspective has to be traced with μ, whose motion is entirely different. Some practice is necessary before being able to handle it successfully.

A certain amount of dead motion is inevitable in an instrument of this kind, particularly when changing the direction of the motion perpendicular to the slides XY and RT. In order to avoid the errors which this would introduce, the pencil M should always be moved in the same direction—away from XY, for instance. When the draughts-

(1) See note, page 77.

man comes to a part of a line or curve which is directed towards XY, he should lift the pencil, push M back to the other end of the curve, and trace it in the opposite direction.

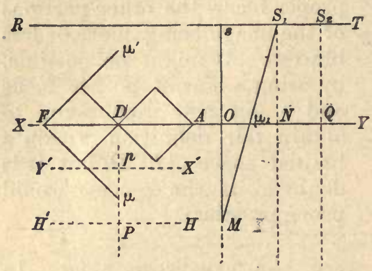

Fig. 144

The position of the horizon line HH' of the perspective (Fig. 144) varies each time the distance from s to S_1 is changed, for it corresponds to the tracer μ when M is at infinitum and the two arms parallel; μ_1 would then be in N. Now, change the height of the station from S_1 to S_2: N comes in Q, carrying with it the joint A of the pantograph system to which it is rigidly fixed; DA is increased by NQ and μ moves down the same quantity. So the horizon line is moved towards the front of the drawing board the same distance as the station is moved up. The ground line is not affected. The instrument is provided with means of adjustment for the distance from A to μ_1, these two points being connected by an iron rod sliding in a ring at A, to which it can be fixed by a clamping screw. A graduation placed alongside the rod permits the increase in the height of the station to be added to $A\mu_1$. This adjustment prevents the displacement of the horizon line. Both the perspective and the ground plan occupy invariable positions on the drawing board, no matter what ground plane may be used.

When the distance line is changed, as for figures in inclined planes, the simplest manner to place the perspective on the board is to put the pencil in M on the trace of the principal plane (Fig. 144), MO being equal to Os; then slide the perspective under the tracer until the latter is on a point midway between the ground and horizon lines. The principal line of the perspective must, of course, coincide with the line previously drawn on the drawing board.

102. Change of scale.—The perspectograph, set as in Fig. 132, will only work on points beyond the picture plane. It is possible, by placing the slide XY on the other side of RT, to reach within the picture plane, but the instrument does not work well. It is preferable to use it as set in Fig. 132, and to resort to a change of scale when figures within the picture plane are to be operated upon.

Let sXY and XYX_1Y_1 (Fig. 145), be the ground and picture planes. Take a new ground plane, $s_1X_1Y_1$, at a distance ss_1 below the

other plane equal to the height of the station Ss; the figures obtained on the new plane from the perspective, will be on a scale double the former scale, (§ 54). The new ground line $X_1 Y_1$ corresponds to the front line AB of the plane sXY, midway between the foot of the station, s, and the picture plane, so that it is now possible to work with the perspectograph on the part of the ground plane comprised between AB and XY, but the result has to be reduced to half size.

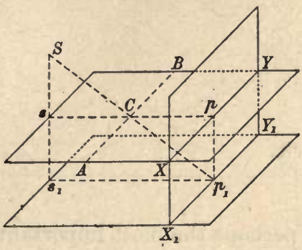

Fig. 145

By doubling the scale again, one half of the space between AB and s is covered; the result must be reduced four times.

In practice, the draughtsman commences by working on the figures beyond the picture plane. After transferring them to the ground plane, he moves S_1, (Fig. 132) so as to double sS_1 and draws a new ground line on the perspective, by doubling the distance of the first one from the principal point. He then places the perspective in proper position for the new ground line and continues to operate the instrument as before.

The ground plan is drawn on cross section paper, having squares of four sizes, distinguished by lines of different intensities. The largest squares are divided into four smaller ones which are also divided into four. The sides of the squares are even divisions of the scale. When the scale is increased two or four times, the reduction is made at sight by transferring the figures from one set of squares to the other. Front lines have been marked on the drawing board at even distances from the foot of the station. The cross section paper is pinned to the board with some of its lines upon the front lines of the board and a perpendicular to the former on the trace of the principal plane.

The distance from the foot of the station is marked on one of the front lines of the paper forming the sides of the squares: this distance, with the trace of the principal plane permits of transferring to the general ground plan the portion of it which has been obtained by the perspectograph.

When the scale is changed, the distance of the front lines should be changed accordingly. Thus, if the scale is doubled, the front line marked 10 on the drawing board corresponds to a real distance of 5 and must be so marked on the paper.

CHAPTER IV.

FIELD INSTRUMENTS.

103. LENSES.—The photographic lens is perhaps the most important part of the instrumental outfit : upon its careful selection depends the perfection and accuracy of the results.

There is no perfect lens ; all are subject to aberrations and deficiencies of various kinds. By combining glasses and lenses of different shapes, the optician corrects some of these imperfections, and it is for the photographer to select from the numerous combinations available that particular one the deficiencies of which will have the least detrimental effect on the work contemplated.

The curves ascribed to lenses are spherical. The line joining the centres of the spheres forming the faces of a lens is the *principal axis.* The lens is *centered* when the principal axis is perpendicular to the planes of the circumferences limiting the faces.

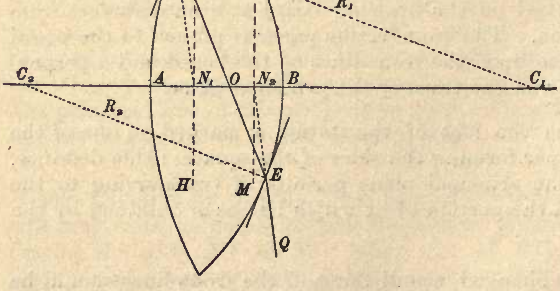

Fig. 146.

Let C_1 and C_2, Figs. 146 and 147, be the centres of the spheres ; R_1 and R_2 their radii. Draw the parallel radii C_1I and C_2E and join IE. The tangent planes at I and E being parallel, the incident ray PI, which is refracted along IE, emerges parallel to its original direction. O is the *optical centre* of the lens ; it is fixed by the relation :

$$\frac{C_1O}{C_2O}=\frac{R_1}{R_2}$$

N_1 and N_2, where the incident and emergent rays intersect the axis, are the *nodal points.* Their position is constant for rays making a small angle with the optical axis. Their distances from the corresponding summits of the lenses are given by the equations

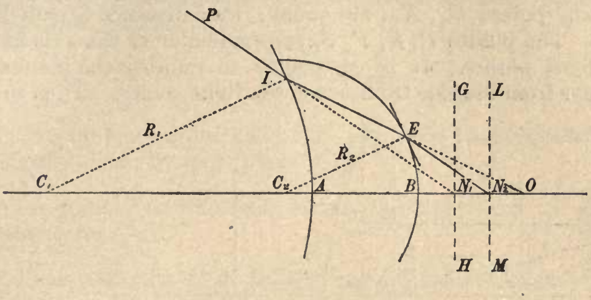

Fig. 147

$$AN_1 = \frac{AO}{n} \times \frac{C_1N_1}{C_1O}$$

$$BN_2 = \frac{BO}{n} \times \frac{C_2N_2}{C_2O}$$

in which n is the refractive index of the glass.

$$\frac{C_1N_1}{C_1O} \text{ and } \frac{C_2N_2}{C_2O}$$

are nearly equal to one: if they are left out and AN_1 and BN_2 designated by a_1 and a_2, AO and BO by q_1 and q_2, the equations become

$$a_1 = \frac{q_1}{n}$$

$$a_2 = \frac{q_2}{n}$$

The distance between the nodal points is:

$$N_1N_2 = \frac{(n-1)e}{n}$$

in which e is the thickness of the lens.

The planes GH, LM, perpendicular to the principal axis at N_1 and N_2 are the *principal planes.*

A small pencil of rays parallel to the axis of a lens, converges to or diverges from a point after refraction through the lens. In Fig. 148, the pencil AG, BH, converges to the point F_2 on the axis. Similarly, a pencil coming from the other direction converges to F_1. These

two points are the *principal foci.* Their distances f_1, f_2, from the corresponding nodal points N_1, N_2, are equal: this distance is called the *focal length.* The planes F_1K, F_2S, perpendicular to the axis at F_1 F_2, are the *focal planes.* It is customary to number the planes and points of a lens from the side from which the light comes. Thus in

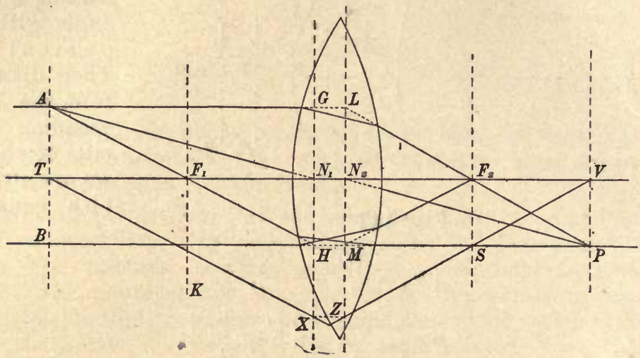

Fig. 148

Fig. 148, the light coming from the left, F_1 is the first principal focus, N_1 the first nodal point, F_1K the first focal plane and GX the first principal plane; F_2 and N_2 are the second principal focus and second nodal point, F_2S and LZ are the second focal plane and second principal plane. In formulas relating to lenses, lengths are considered positive in the direction of the light's course and negative in the opposite direction. In Fig. 148, the light coming from the left, the focal length, F_1N_1 or f_1 is negative and the focal length F_2N_2 or f_2 is positive. In Fig. 146, the radius R_1 is positive and R_2 is negative. In Fig. 147 both R_1 and R_2 are negative.

The value of the focal length is

$$f_2 = -f_1 = \frac{R_2 \; R_1}{(n-1) \left[R_2 - R_1 + \frac{n-1}{n} e\right]}$$

n being the refractive index of the glass and e the thickness of the lens. A positive value of f_2 indicates that parallel rays converge to the principal focus after refraction; a negative value of f_2 indicates that the rays diverge from the principal focus.

In thin lenses, where e can be neglected, the formula becomes:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The average value of n is 1·5; it gives for the approximate value of the focal length:

$$\frac{1}{f}=\frac{1}{2R_1}-\frac{1}{2R_2}$$

An oblique pencil AF_1, TK (Fig. 148) converges after refraction to a point S in the focal plane F_2S.

A point T on the axis forms its image at V; V and T are called *conjugate foci.*

The point A and its image P are called *conjugate points.*

Designating by f_1 and f_2 the distances TN_1 and VN_2, we have:

$$\frac{1}{f}=\frac{1}{f_2}-\frac{1}{f_1}$$

Also:

$$f^2=-\varphi_1\varphi_2$$

in which φ_1 and φ_2 are the distances TF_1 and VF_2, from the object and its image to the principal foci.

The image P of any point A in a plane AT perpendicular to the axis is in the conjugate plane VP.

The intersections X and Z of an incident ray TX with the first principal plane and of the refracted ray ZV with the second principal plane are on a parallel to the principal axis.

The above definitions hold good only for lenses of small aperture and for rays making small angles with the optical axis. When these conditions are not fulfilled, discrepancies occur which are called *aberrations.*

A lens is fully defined by its principal planes and foci. When these are known, the course after refraction of any ray or pencil of rays is easily found.

To find the image of a point A, join AF_1 and produce to the intersection with the first principal plane at H; this ray passing through the first focus is, after refraction, parallel to the axis in HP.

Draw AL parallel to the axis until it intersects the second principal plane in L. This ray being parallel to the axis, is refracted in LF_2 to the second focus.

Join AN_1: the refracted ray is parallel to its original direction and passes through the second nodal point N_2.

The image P of A is at the intersection P of the three refracted rays.

The course of any ray, TX, after refraction, is found by considering a parallel ray F_1H through the first focus. Its course after refraction is in HS parallel to the axis. A pencil parallel to F_1H forms its image at S in the second focal plane; TX must therefore after refraction converge to S. As the intersections of the incident and refracted rays with the principal planes are on a parallel to the axis, the course of the refracted ray is obtained by drawing XZ parallel to F_1F_2 and joining ZS.

The conjugate foci T and V are fixed in the same manner.

The rays, of course, follow these directions only before entering the lens and after emerging therefrom.

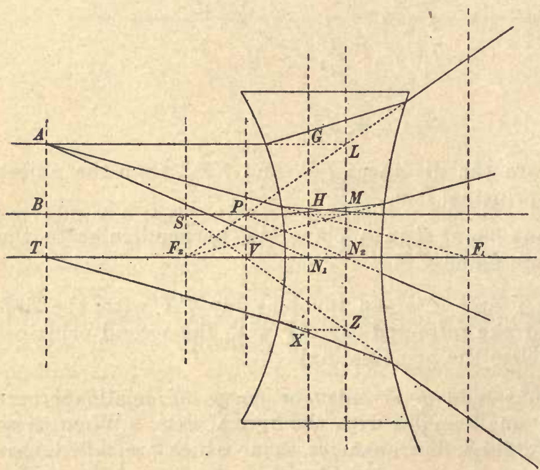

Fig. 149

Their course inside of the lens is shown by solid lines in the figure.

These constructions and definitions apply also to negative lenses, but in the latter, a pencil parallel to the axis diverges from the principal focus after refraction instead of converging to it.

In Fig. 149, the same constructions have been made and the same letters used as in Fig. 148. With the positive lens the image P of the point A is real: with the negative lens, it is virtual. A point between a positive lens and its principal focal plane also gives a virtual image.

To find the principal and focal planes of a combination of any number of lenses, a ray parallel to the axis is supposed to enter the lenses on one side; its course is constructed as already explained, and the point where it intersects the axis after the last refraction is one of the principal foci of the combination. The corresponding principal plane is given by the intersection of the original and final directions of the ray. The same process applied to a ray parallel to the axis and coming from the opposite direction, gives the other focus and principal plane.

Fig. 150 illustrates the combination of a positive lens $F_1\ N_1\ F_2\ N_2$ and a negative lens $F'_1\ N'_1\ F'_2\ N'_2$. The ray AB parallel to the axis

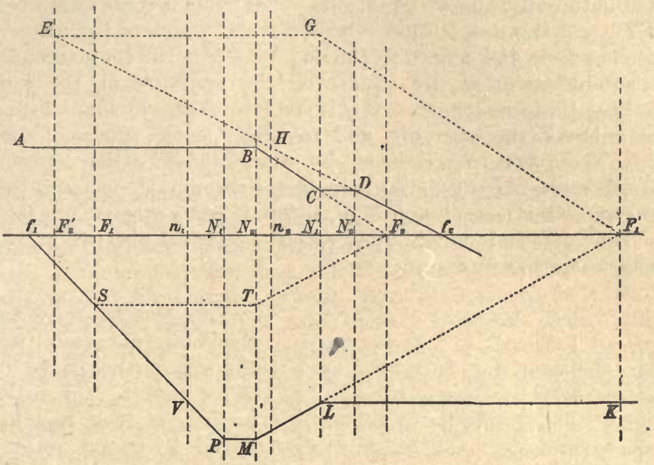

Fig. 150

intersects the second principal plane of the first lens at B and is refracted in the direction of the second focus F_2; it then intersects the first principal plane N'_1 of the second lens at C, and therefore must be refracted in a direction passing through D in the second principal plane N'_2 on the parallel CD to the axis. GF'_1 is drawn parallel to BF_2 and EG parallel to the axis; ED is the direction of the refracted ray and f_2 the second focus of the combination. The intersection H of the original and refracted rays is a point of the second principal plane n_2.

By the same process the ray KL gives the first focus f_1 and the first principal plane n_1.

104. Aberrations.—The lenses hitherto considered had small apertures and the pencils of light were not much inclined on the axis. When the conditions are different, the rules given above are only approximate. A pencil parallel to the axis does not, after refraction through a single convex lens, converge exactly to one point. The central rays meet

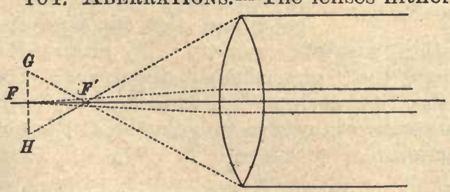

Fig. 151

in F (Fig. 151), and the marginal rays in F'. In trying with this lens to focus a luminous point on the ground glass of the camera, it is impossible to obtain good definition. The focus is somewhere between F and F'', and the image appears as a circular patch of light decreasing in intensity from the centre to the edge. FF' is the longitudinal aberration; it increases as the square of the aperture of the lens and inversely as its focal length. GH is the lateral aberration: it increases as the cube of the aperture and inversely as the square of the focal length. The aperture is reduced by diaphragms: their effect is to cut off the marginal rays. In focussing a camera, the general practice is to do so with a large diaphragm, and to insert a smaller one for exposing. When there is much uncorrected spherical aberration, the two foci may be quite different.

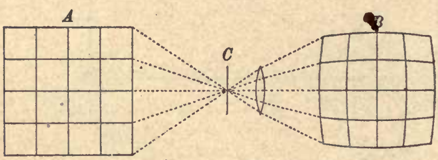

Fig. 152

The greater refraction of the marginal rays towards the axis is the cause of another imperfection, called *distortion*, in the images produced by single lenses with diaphragms. Fig. 152 shows the diaphragm C in front of the lens: the square grating A, perpendicular to the principal axis, gives an image like B with "barrel-shape" distortion. The axial rays are not displaced, but the other rays are refracted more and more towards the axis as they approach the edge of the lens. The single rays represented may be taken as the axes of pencils. The "pin-cushion" distortion occurs when the diaphragm is behind the lens (Fig. 153). The marginal rays are still more refracted than the central rays, but after they have crossed the axis at the diaphragm, they become divergent and the opposite distortion occurs. This explanation suggests a remedy: by placing the diaphragm between two lenses, the "pin-cushion" distortion of the front one is compensated by the "barrel-shape" distortion of the back one. Although this is the simplest form of a rectangular lens, it is not the only way to correct distortion.

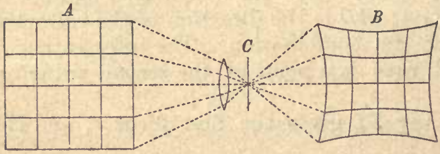

Fig. 153

In passing through a single lens, the different rays of which white light is composed are unequally refracted, the deflection being greater for the violet than for the red part of the spectrum. With a pencil of

white light the violet rays come to a focus at V (Fig. 154), the green rays at G and the red rays at R. At none of these points is a sharp or uncoloured image obtained. This is called *chromatic aberration.* The brightest part of the spectrum is near the line D, and in adjusting a camera, the best image is seen on the ground glass when these rays are in focus. On the other hand, the chemically active part of the spectrum on ordinary plates is about the lines G and H, so, with an uncorrected or undercorrected lens, the dry plate requires to be placed at some distance in front of the ground glass in order to be in the focus of the blue violet rays. The lens has a "*chemical focus*" different from the visual focus. Chromatic aberration is corrected by combining lenses made of glass of different refractive and dispersive powers. A positive lens of small dispersive power associated with a negative lens of great dispersive power can be made achromatic although still remaining convergent. With two kinds of glass, two lines of the spectrum may be brought to the same focus, but some slight discrepancies remain for the other rays and form secondary spectra. If n kinds of glass are employed, n lines can be brought to the same focus. Photographic lenses are corrected for the G or H lines, which are the most active rays on ordinary dry plates. It will be shown later that some orthochromatic plates exposed behind deep orange glass are sensitive to yellow rays only; the chromatic correction of photographic lenses is, therefore, not as good as it might be for these plates.

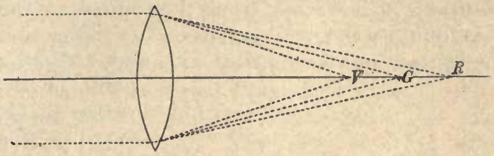

Fig. 154

In focussing a figure AB in a plane perpendicular to the optical axis EF, (Fig. 155), different positions of the ground glass will generally be found for the centre F, and the sides C and D of the figure; this indicates *curvature of the field.* A diaphragm GH reduces the curvature by cutting off the rays of shorter foci. The farther the diaphragm is from the lens, the flatter the field becomes but the distortion is increased.

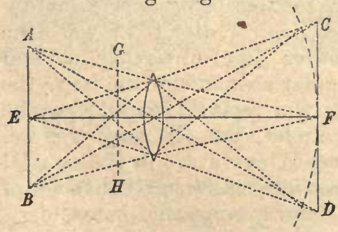

Fig. 155

Another difficulty is frequently experienced in focussing extra axial points. In a position of the ground glass, the image of a luminous point is a short line directed towards the centre of the field; in another position, the image is also a short line but perpendicular to the first one. Between the two positions, the images are patches of light elongated in a radial or tangential

direction, according as the ground glass is nearest to the first or last position. This is *astigmatism;* it increases from the centre to the margin of the field. It is due to the convergence after refraction being smaller in the plane containing the optical axis and radiating point, or meridian plane, than in the perpendicular or sagittal plane. Let AC GK (Fig. 156) be a small rectangular element of a lens on which falls a pencil of parallel rays; BH being the section of the lens by the meridian plane and DE its section by the sagittal plane. The meridian plane containing the centres of the spherical faces, its intersections BH by the latter are great circles of the spheres. The intersections of the parallel planes AG and CK are very nearly great circles, because they are at very short distances from the meridian plane. The sagittal intersections AC, DE, GK are small circles of the spheres; the curvature of the faces is thus greatest in the sagittal plane and smallest in the meridian plane. The convergence of the rays D, O and E is therefore greater than the convergence of B, O and H, the first ones meeting in F' while the latter intersect at F. The convergence in the planes of parallel sections being approximately the same, there are formed two focal lines, one, F', directed toward the centre of the field or radial, and another one, F, farther off and tangential. In the other small rectangular elements of the lens, the sagittal sections are still small circles; the sections parallel to the meridian plane are not exactly great circles, but the same general character is preserved throughout of greatest curvature in the sagittal sections and smallest in the meridian sections, each small element of the lens giving in the same way a radial and a tangential image.

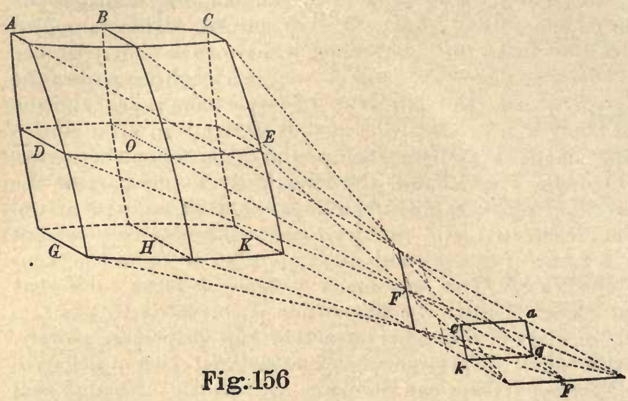

Fig. 156

There are thus two focal surfaces tangent on the optical axis, and becoming more and more separated as the distance from the centre increases. The length FF' is the astigmatic difference. In focussing on a radial line, the best definition is at F'; on a tangential line, it is at F. The best mean focus is somewhere between the two, *acgk* being the image of the radiating point and the figure of least confusion.

Some lenses give what is called "*a flare spot*"; it is a circular patch of light, more or less intense, in the middle of the image. J. H. Dallmeyer found by experiment that it was an image of the opening in the diaphragm caused by the back lens of combinations, but he could not understand how a real image was formed with such a short distance between the lens and the diaphragm. The explanation was given by Sir John Herschell. The rays entering the back lens are not all refracted; some are reflected by the back surface, and reflected again by the front surface and it is by these rays that the image of the diaphragm's opening is formed. In lenses of good construction, the curves of the surfaces are calculated to throw the greater portion of the reflected rays outside of the field and to spread evenly over it the remaining ones.

105. DIAPHRAGMS.—The aberrations increasing rapidly with the aperture of a lens, the use of diaphragms naturally suggests itself. In a single lens they are placed in front, because the barrel shape distortion which they produce is less unsightly than the spindle shape. In combinations, they are between the lenses.

The brightness of the image depends on the quantity of light admitted by the lens, and this is proportional to its aperture or to the square of its diameter. The larger the aperture, the more light is admitted. The brightness further varies inversely as the square of the focal length; for instance by doubling the focal length, the dimensions of the image are doubled and the light admitted through the lens is distributed over an area four times larger. The brightness of the image is reduced in proportion.

Representing by d the diameter of the aperture, and by f the focal length, the brightness is proportional to

$$\frac{d^2}{f^2}$$

This fraction is the measure of the rapidity of a lens, and the number by which the camera exposure for different lenses or diaphragms is computed. There are, however, several allowances to be made. A portion of the light is lost by reflection on the surface of the lenses, and some is absorbed by the glass. If an exposure of 100 seconds be required with a single lens and if each lens absorbs or reflects one-tenth of the light, then the proper exposure with a double combination, such as an aplanat, is 111 seconds; with a triplet, it is 123 seconds. The colour of the glass has a great influence on the rapidity of a lens.

The breadth of the pencils admitted by the diaphragm diminishes with their inclination to the axis; q being the area of the section of the axial pencil, the section of a marginal one making an angle α with the axis is:

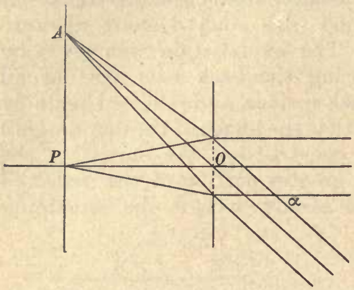

Fig. 157

$$q \cos \alpha$$

But an object forms an image larger in A (Fig. 157) than in P, the proportion being:

$$\frac{AO}{OP} = \frac{1}{\cos \alpha}$$

The proportion of the areas is:

$$\frac{1}{\cos^2 \alpha}$$

and as the image at A is formed by pencils of reduced section, its brightness, taking as unit the middle of the field, is

$$\frac{1}{\cos^3 \alpha}$$

For several reasons the decrease in illumination from the centre to the margin of the field is in reality larger than this. It is well marked in photographs taken with wide angle lenses; the edges are always much darker than the middle.

The exposure required by the photographic plate is inversely proportional to the brightness of the image formed on its surface; it increases in the ratio:

$$\left(\frac{f}{d}\right)^2$$

It is important that each diaphragm should be numbered to indicate its rapidity.

In the "uniform system," the exposure for a diaphragm of an aperture of $\frac{f}{4}$ is taken as unit, and each diaphragm is marked with the corresponding exposure as follows:—

$$\text{No. 2 for } \frac{f}{5\cdot66}$$

$$\text{" 4 " } \frac{f}{8}$$

$$\text{" 8 " } \frac{f}{11\cdot31}$$

$$\text{" 16 " } \frac{f}{16}$$

$$\text{" 32 " } \frac{f}{22\cdot6}$$

Zeiss takes as unit $\frac{f}{100}$: his numbers are:

$$\text{No. 1 for } \frac{f}{100}$$

$$\text{" 2 " } \frac{f}{71}$$

$$\text{" 4 " } \frac{f}{50}$$

$$\text{" 8 " } \frac{f}{36}$$

$$\text{" 16 " } \frac{f}{25}$$

$$\text{" 32 " } \frac{f}{18\cdot5}$$

$$\text{" 64 " } \frac{f}{12\cdot5}$$

$$\text{" 128 " } \frac{f}{9}$$

$$\text{" 256 " } \frac{f}{6\cdot3}$$

Dallmeyer adopts for his unit

$$\sqrt{\frac{f}{10}}$$

the International Congress of Photography has recommended

$$\frac{f}{10}$$

and several opticians have systems of their own. Many mark on their diaphragms nothing but the value of

$$\frac{f}{d};$$

that is not, perhaps, the most convenient system, but it is the least liable to cause confusion.

The section of the axial pencil passing through the lens is equal to the opening of the diaphragm when it is in front of the lens: d is the diameter of the aperture and may be measured with a scale. When a lens or lenses are in front of the diaphragm, the effective and real apertures are no longer equal. The axial pencil converges after refraction to the focus F, Fig. 158, of the positive front combination, and its section is reduced when it reaches the diaphragm. The diameter of the effective aperture is:

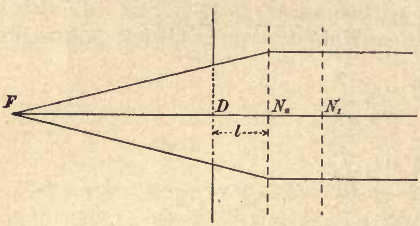

Fig. 158

$$d' = d\frac{f}{f-l}$$

d is the diameter of the opening, f the focal length of the front lens or combination, and l the distance from its second nodal point to the plane of the diaphragm.

The effective aperture d' and the co-efficient

$$\frac{f}{f-l}$$

may be found by the following process due to Steinheil. After focussing on a distant object, replace the ground glass of the camera by a screen with a small hole in the middle. Place a bright light close to and behind the hole. Cover the lens with a ground glass; an illuminated circle is seen on it which is the effective aperture. The diameters of this circle and of the diaphragm are measured; their ratio is the co-efficient by which d must be multiplied.

106. Anastigmats. The lens of a surveying camera must be free from distortion ; it must cover an angular field of about 60°, and the definition must be uniform all over the plate. Rapidity is not essential. These conditions restrict the choice to what are known as "wide angle lenses." They are doublets consisting of two lenses placed close together and between which is the diaphragm. The lens adopted for the Canadian camera is Zeiss' anastigmat of $f/18$ aperture and 141 millimetres focus. Fig. 159 represents the lens full size. The radii, R, thicknesses, e, and refractive indices for the line D of the spectrum, n, are as follows :—

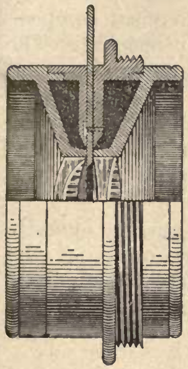

Fig. 159

FRONT LENS.

$n = 1{\cdot}55247$
$R_1 = +13{\cdot}14$ millimetres
$R_2 = +6{\cdot}61$ "
$e = 1{\cdot}25$ "

$n = 1{\cdot}51720$
$R_3 = +6{\cdot}61$ millimetres
$R_4 = +14{\cdot}71$ "
$e = 1{\cdot}76$ "

BACK LENS.

$n = 1{\cdot}51674$
$R_5 = -28{\cdot}33$ millimetres
$R_6 = +20{\cdot}13$ "
$e = 1{\cdot}00$ "

$n = 1{\cdot}57360$
$R_7 = +20{\cdot}13$ millimetres
$R_8 = -28{\cdot}33$ "
$e = 2{\cdot}89$ "

The axial distance between the two lenses is 2·88 millimetres. The diaphragm is midway between the lenses. The special features of the anastigmats cannot be illustrated better than by giving the following extracts from a lecture by Dr. Paul Rudolf at the Photographic Convention of the United Kingdom ;—*

The Zeiss anastigmatic lenses are dissymmetrical doublets consisting of an achromatic anterior part, whose flint has the higher refractive in-

*British Journal of Photography, 28th July, 1893.

dex, and an achromatic posterior part in which the crown has the higher refractive index. These two cemented parts of the doublet possess, therefore, opposite differences of refractiveness in the crown and flint glasses employed for achromatisation. "Crown" and "flint" are here placed in opposition, not with respect to their chemical composition, but are considered with respect to their optical properties. The same glass may therefore appear in two different achromatic combinations either as crown or flint. In the following remarks, crown glass is always understood to refer to that glass of a binary lens which is less in relative dispersion, while the term flint glass refers to that glass which has the greater relative dispersive power.

The combination of the two cemented parts having opposite differences of refractiveness in the crown and flint, embodies the important principle by which it became possible to effect anastigmatic aplanatism of a system of lenses corrected spherically and chromatically for large apertures.

The series of new glasses produced by the works of Messrs. Schott & Co., of Jena, rendered it possible to realize this principle in the construction of Zeiss anastigmatic lenses.

Refractive and dispersive powers were, with the older glasses, dependent upon each other in a certain manner; an increase in the one corresponded to an increase in the other. In order that anastigmats may be constructed, it is, however, necessary that a range of glasses be obtained in which any refractive index may be coupled with any desired dispersion. This is accomplished by the glasses made by Messrs. Schott & Co.

All photographic lenses preceding the anastigmats had either a very much curved field, or, if flat, an astigmatic field. In the latter case the image was perfect in the centre, but deteriorated towards the edge.

It may be useful here to define what is meant by a curved, flat and astigmatic image.

Let the object be placed in a plane which is accurately normal to the axis of the objective, or let the object be at so great a distance from the objective that the difference in the distance of the different points may be neglected. Then the field of an objective is considered to be curved if different positions of the focussing screen are required for sharply focussing a point in the axis (centre of the image) and a point lying outside the centre of the image.

The whole of the older photographic lens types exhibit curvatures of the image, such as to necessitate the distance of the focussing screen being shorter for marginal points of the image than for axial points. The points of distinct delineation are, therefore, situated upon a curved surface whose concave side is presented to the objective. Fig. 160 represents the section of the image surface, and a plane passing through the axis of the objective. The dotted line *a* represents the ideal image plane, which intersects the axis in the axial image point, and is at right angles to the axis of the objective, while curve *b* represents the actual surface of the image.

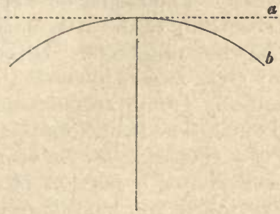

Fig. 160

The field is flat if the position of the focussing screen is the same for central and extra-axial points—*i.e.*, if the sharply focussed points are all contained in a plane which is at right angles to the axis of the objective, or if these points lie in the ideal plane of the image.

The image is astigmatic if sharp images of lateral, *i.e.*, extra-axial, points may be obtained by two different positions of the focussing screen. The two images are not exactly similar to the object; one of them shows distortions in the directions radiating from the axis (radial distortion), while the other exhibits distortions in the directions at right angles to the radii (tangential distortion). This fact may easily be demonstrated by means of an "aplanatic" lens, say, by using a small circular disc having a diameter of only a few millimetres situated outside the axis and attempting to sharply focus it. With the nearer position of the focussing screen, the image of the disc appears as a radial line of a breadth proportional to the diameter of the disc. With the longer distance of the screen, the image is an arc of a circle concentric to the axis of the objective (tangential distortion). The lengths of the radial and tangential portions of the line are essentially dependent upon the difference of the two positions of the screen, and increase continuously from centre to margin in the case of the "aplanatic" lens. With objectives yielding astigmatic images, there are thus two image surfaces conjugate to one and the same object plane. These two image surfaces touch each other in the axial image point, and the distance between them increases continuously from centre to margin. "Mean curvature" may be defined as that surface which represents the arithmetical mean of the deviations of the two image surfaces from the ideal surface. The dissimilarity between the details in the two image surfaces inter se and the original, increases from the centre to the margin. The following is an interesting experiment:—

Fig. 161

Arrange in one plane (Fig. 161), along the radii of concentric circles, bright discs. The angles between the radii should be chosen according to the astigmatic aberrations and the focus of the objective. Direct the axis of the objective at right angles towards the centre of the system of radially grouped discs, and focus one of the extra-axial discs. The image obtained at the shorter distance of the focussing screen from the objective is, as Fig. 162 shows, a portion of a radial line which, in proportion to the curvature of the image plane, becomes more and more indistinct towards either side, and is more or less interrupted radially in proportion to the degree of astigmatic deviation. Fig. 162 is an image obtained by focussing a disc on a circle situated midway between the axis and the outside circle. It will be seen that images of all the discs grouped along the same circle are of the same degree of distinctness or indistinctness, and also exhibit the same amount of distortion.

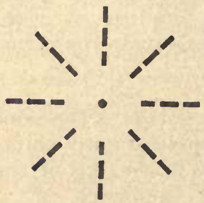

Fig. 162

Focussing at the greater distance of the screen shows the object (Fig. 163) tangentially distorted. The image thus becomes composed of a series of circles concentric to the centre of the image, which are more or less interrupted, or perfectly uninterrupted. The discs grouped along another circle, which has not been sharply focussed, are similarly distorted, though in a less marked degree.

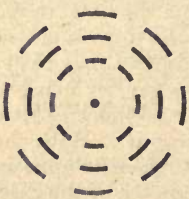

Fig. 163

If, now, a screen having drawn upon it circles concentric to the axis of the objective and radii, be substituted for the system of discs, the astigmatic objective would reproduce the original with partial similarity, but both systems of lines could not simultaneously be delineated with the same position of the screen. The shorter distance would yield sharpness of the radial elements, the circles at the same time being badly defined ; the longer distance would show the circles sharply, and at the same time the radial elements badly defined.

A sufficiently large screen bearing the two systems of lines, radial and concentric circular lines, appears thus to form the most natural test for astigmatism. Such a screen would, however, be too uniform and too little adapted for exhaustive tests.

The screen 2 x 2 metres area, as it is used in the photographic laboratory of Carl Zeiss, of Jena, is for this reason arranged somewhat

differently, and it may not be uninteresting to give here a short description of it. Upon strips of paper of 18 x 21 cm. in area are two systems of parallel lines of varying thickness crossing each other at right angles and placed at varying distances; the strips themselves are fixed to the screen with one of their ends at the centre of the screen, and their sides parallel to the sides of the screen in such a manner that in each strip the radial and tangential lines alternate. The middle of the screen contains a field consisting of rectangular cross lines, which is intended for testing sharpness of definition. The tangential parallels form substitutes for the system of concentric circles, while the radial parallel lines take the place of the radii proceeding from the centre. The difference between the two positions of the focussing screen for sharp delineation of the marginal portions of the systems of straight lines represents the astigmatic difference. For the purpose of demonstrating the incorrectness of the image caused by astigmatism, the screen has square fields of more or less fine rectangular cross lines, diagonally attached to it in such a manner that, in one the system of lines is parallel and at right angles respectively to the diagonal, while in the other case they are inclined at 45° to the diagonal. The screen has also samples of writing and printing attached to it.

When focussing square cross lines at the edge of the image, the astigmatic objective produced, in the two characteristic cases furnished by the test screen, the following deformations :

1. The straight lines composing the net at the edge of the image are parallel and at right angles respectively to the direction of the radial lines.

In this case—represented by Fig. 164,—in which A is the point of intersection of the axis of the objective and the plane of the object, A' that of the axis and the plane of the image—sharp focussing of the tangential lines causes the lines which are at right angles to the radius to appear nearly sharp, while the lines which are parallel to the radius are almost entirely invisible (image a). Focussing of the radial lines produces the converse of the last test. The lines parallel to the radius appear sharp, the lines at right angles to it disappear (image b). Mean focussing results in a totally ill-

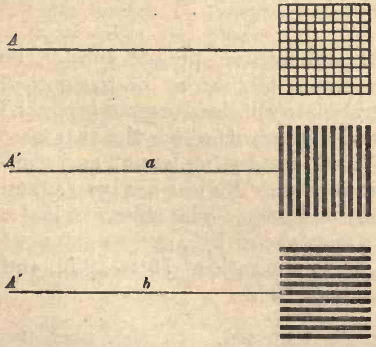

Fig. 164

defined image, and eventually in more or less marked reversion of the cross lines, such as a white net in a black field.

2. The straight lines of the net are inclined at 45° to the radial direction. In Fig. 165, let *A* and *A′* again be the points of intersection of the objective axis with the planes of the object and image respectively. Tangential focussing causes the rectangular cross lines to be distorted so as to present the appearance of tangentially elongated hexagons, and, in the case of great astigmatic difference, it may result in almost precise commutation of the cross lines into a single system of tangential lines (image *a*, Fig. 165.)

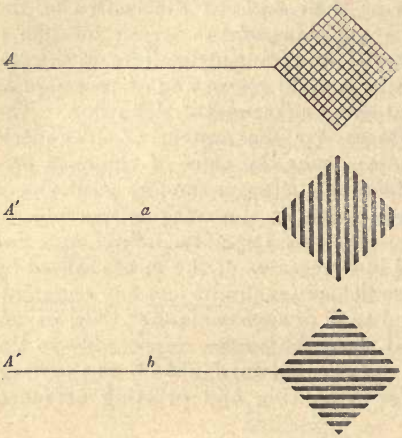

Fig. 165

Radial focussing produces radially elongated hexagons and may, with great astigmatic difference, result in changing the cross lines into a single system of radial lines (image *b*, Fig. 165.)

If we focus between these two limits, the net may, similarly as above, eventually be reversed so as to appear as black points in a white field ; the same effect may also be produced in anastigmatic images by unsharp focussing. Similar results of a more or less marked character may be obtained by replacing the quadratic net by one formed of oblongs, rhombi, circles, &c.

In order that these relations might be illustrated, photographs of the test screen were taken in the photographic laboratory of the optical works of Carl Zeiss, and the photographs so obtained were reproduced by photo-lithography. There are four plates, of which the two most characteristic ones are Nos. I and IV. Here an "aplanat" and "anastigmat" are subjected to direct comparison. Plate I has been taken with an "aplanat" made by a renowned firm. The objective had a focal distance of 14 cm. and a relative aperture of F/6, and was stopped down to F/12·5. Image and object are in the ratio of 75 to 1000, and the angle subtended by the object is about 67°.

The centre of the screen is sharply focussed. In this part the delineation is extremely good, a sufficient proof that the objective, per se, was a good specimen of its kind. As the margin is approached,

the definition, however, loses more and more in distinctness, and astigmatic distortion increases more and more. While the tangential lines are fairly sharp up to the edge, the radial lines rapidly decrease in definition past the third part of the field. In the diagonal squares, the bounding lines of which are at right angles and parallel respectively to the radius, it is noticed that the tangential lines are markedly sharper than the radial lines, the latter being almost invisible, and in the squares, whose sides are inclined at 45° to the radial direction, the distortion at the margin of the tangential lines may readily be observed. The squares appear, in fact, as hexagons.

Plate IV was taken with a Zeiss anastigmat, viz., anastigmat F/6·3 of 14 cm. focus, all other conditions being the same as those existing in the former case. There, too, the centre was accurately focussed, but barely any traces of those details which point to astigmatic imperfections of the margin of the image will be noticed.

Plates II and III were taken with the same "aplanat" as that used for Plate I. In the first case the tangential marginal lines were focussed; in the second case the marginal radial lines formed the critical part of the object. While in the former case the centre appeared to be fairly sharp, in the latter case it was totally worthless. The characteristics of astigmatism, as above explained, become apparent in both plates.

The older types of lenses (aplanatic, antiplanats, portrait lenses, single lenses, &c.) admitted of astigmatic correction, but they could not at the same time be corrected for flatness of field. The Zeiss "anastigmat" was the first lens in which, as we pointed out, anastigmatic aplanatism was combined with the realization of other requisites of a good photographic lens.

A lens having anastigmatic curvature yields sharply defined points from centre to margin. These cannot, however, simultaneously be fixed upon the plane negative plate of the photographic apparatus when the points constituting the object are nearly in one plane at right angles to the axis, or when they are at a relatively great distance from the objective. If it were desired to simultaneously fix these sharp points upon the plate, it would be necessary to use a curved sensitive surface corresponding to the curvature of the image. Clearly the use of such curved sensitive strata is impossible, for it must not be forgotten that for each lens type, each focal length, and even each degree of magnification or reduction, there is a distinct corresponding curvature, to say nothing of the practical inconvenience attached to curved photographic plates. At present we are, at any rate, limited to flat negative plates. From an optical point of view this is an undesirable limitation, which seriously affects definition and depth of the curved image.

The flat plate cannot be covered uniformly and sharply from centre to edge, unless the objects are grouped on a curved surface corresponding to the curvature of the image. With portrait groups, photographers have a means of compensating the anomaly by arranging the persons in a semicircle, in the centre of which the objective is placed. With landscape and instantaneous photographs, however, such an expedient is only rarely, if at all, applicable. In order to obtain tolerable distinctness in the image from centre to edge it would be necessary to work with narrow angles or to stop the lenses down considerably.

In working in this manner it must be borne in mind that both in the centre and at the edge, near and far objects are to be depicted simultaneously; the objective, yielding a curved image, causes, however, distinct objects on the photographic plate to appear indistinct, and sufficiently near objects sharp at the edge when the focussing is sharp for the centre. On the oblong flat negative plate *ABCD* (Fig. 166) let near objects be depicted at *AB*, distant objects at *CD;* then, if the centre, *M*, be accurately focussed, the points of sharp delineation are situated upon a curve, and are represented by *E*, which intersects *AB* in two points, and is symmetrical with respect to *AB*. By stopping the lens down we obtain, as is well known, greater depth for distant objects when focussing for near objects. In the present case the depth necessarily diminishes from the centre to the margin. The limits may be graphically represented by curves T_1 and T_2, which, being symmetrical with respect to *E*, have their greatest distance apart at *M*. The depth of focus is represented by the area contained between T_1 and T_2. By this area the imperfections of the marginal image may readily be ascertained. When it is important to improve the distinctness at the edge it is necessary to sharply focus a point situated at a distance from the centre and to sacrifice the distinctness at the centre.

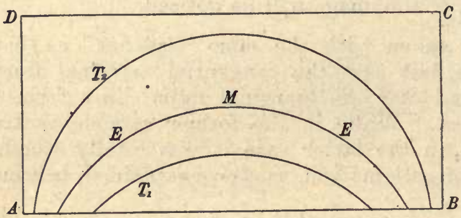

Fig. 166

The deficient depth of focus of a lens yielding a curved image does not, under certain conditions, become apparent in street scenes. In such cases it may happen that the position of the camera is such that the rows of houses are delineated simultaneously on both sides of the street, the distant houses being thus shown in the centre, the near ones at the edges of the plate. In such a case the curvature of the

image may even become the very cause of greater marginal distinctness than is obtainable with the flat field. But distant objects have nearly always to be shown simultaneously at centre and edge, and in such cases it is absolutely necessary to have a flat field.

It is possible to partly flatten the field of the aplanat. This is most conveniently done with those points of the image which are due to the meridional rays, *i. e.*, for the tangential directions in the image. Under these circumstances, one would, however, have to abandon the anastigmatic correction of the image, and to rest content with partial distinctness. Those points of the image, which are due to rays contained in a sagittal section, yield another image surface (image points of radial directions), which touches the former surface in the axis of the objective, and deviates from it with continuous curvature towards the edge, as already explained.

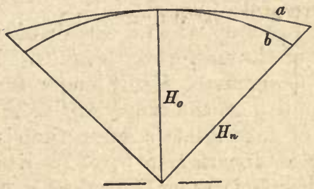

Fig. 167

With angles of 50°, this deviation amounts to one-fifteenth; with 70°, to one-sixth; and with 90°, nearly to one-third of the focal length of the lens. The section of these image surfaces by a plane passing through the axis of the lens would present the appearance shown in Fig. 167. Curve *a* appertains to the image points in the meridional section (tangential distortion), *b* to those of the sagittal section (radial distortion). H_0 represents the axis of the lens, H_n a secondary axis.

This result may be obtained with aplanatic lenses if their halves be sufficiently widely separated. The marginal distinctness is then very defective, and the stopping down has to be carried very far if it is at all desired to obtain sharp definition at the edge or fair definition extending over a considerable field.

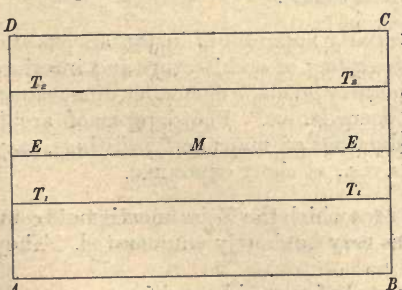

Fig. 168

An objective having an anastigmatically flat field, such as the "anastigmat," produces, however, a sharp image upon the flat plate, which, as Fig. 168 shows, is bounded by lines T_1 T_2 parallel to the focussing line *E*. This objective delineates near and distant objects with the desired uniform sharpness at centre and edge.

The Zeiss anastigmats yield, therefore, a uniform depth of focus from centre to margin without

necessitating the same amount of stopping down that is imperative with the "aplanats." The anastigmats have in proportion to their covering power a considerable relative rapidity.

Owing to the better concentration of light in the anastigmatic flat image, as compared with the anastigmatic curved or astigmatic flat images with an objective of the former type, the intensity necessarily diminishes less from centre to edge than with a lens belonging to either of the last named types. This advantage of the Zeiss anastigmats cannot be overrated, as the oblique incidence of rays at the edge of the image is, in itself, productive of a continuous diminution of intensity towards the edge. The anastigmats yield a negative which is uniformly exposed from centre to margin. The advantages resulting from the anastigmatic flatness of field greatly extend the range of the applicability of these lenses.

The advantage of being able to use a large stop when a certain size of plate is prescribed, and the advantage of the uniformly bright field assist in the solution of the problem of using short focus lenses for relatively large plates. With a given rapidity of the objective, essentially shorter foci may be used in the case of anastigmatic lenses than is admissible with other types. For instance, anastigmat F/6·3 (Series II) of a focus of 105 to 120 mm. is quite sufficient for sharply covering a plate 9×12 cm. ($3\frac{1}{2} \times 5$ inches) at F/9; with the older types the focus would have to be 190 mm. ($7\frac{1}{2}$ inches) at least. In order to cover 13×18 cm. (5×7 inches) at F/9, it was necessary to employ a lens of, say, 350 mm. (14 inches) focus, whereas, with the anastigmats this result can be obtained with a focus of 210 mm. ($8\frac{1}{2}$ inches), and even with 170 mm. (7 inches). Short foci give, however, at equal distances of the object, a better depth than long foci; they yield a sharper image of objects situated at different distances from the apparatus. The anastigmatic lenses have, therefore, in another sense, greater depth of focus than the older lenses.

These advantages become particularly apparent in instantaneous and wide-angle lenses, and in the photography of architecture and interiors and in copying. Detective cameras may be made of smaller dimensions, as they may be fitted with short-focus lenses. Photographs of architecture and interiors, and reproductions of maps and paintings, may be taken by means of rapid lenses, *i.e.*, at short exposures.

In conclusion, the other advantages which the Zeiss anastigmats combine with the anastigmatic flatness may be shortly enumerated. They are as follows:—

1. The reflection images have a most favourable position.
2. They admit of the most colourless glasses being used; and
3. The two parts of the doublet are in close proximity.

The images formed by reflection at the boundary surfaces between glass and air are all at a considerable distance from the plane of the image. By this means the appearance of fogged images, which generally increases with the number of isolated lenses, is reduced to a minimum, and thereby the image rendered exceedingly brilliant.

The existence of this property is amply proved by photographs taken with the anastigmatic lenses.

None of the anastigmatic lenses can be shown to have a flare-spot, even when dazzling light enters the objective.

The use of colourless glasses is an advantage which cannot be overrated. Apart from sensitive plates, this is the only means of satisfying the universal postulate, depth of definition with short exposures.

With objectives of the same type, a certain desired amount of depth can, with a given focal length, only be obtained by corresponding stopping down of the lens. The further, however, this stopping down is carried, the less becomes the light which can pass through the lens. If, in addition to this, the scanty light thus admitted is further impaired by detrimental colouring in the glasses, as was the case with the glasses formerly used in the construction of aplanats, it becomes naturally impossible to work at short exposures.

The anastigmats, when applied to outdoor photography at F/18, give fully exposed negatives, the usual commercial instantaneous dry plates being used. Before the application of the Schott baryta glasses to the construction of photographic lenses, this belonged to the province of impossibilities. Even with stops F/25 and F/36 instantaneous photographs are still obtainable.

The short structure of the anastigmats restrains the diminution of the rapidity with which the intensity decreases from centre to edge. It diminishes that part of the decrease of the intensity which is caused by partial stopping of the pencils by the edges of the lenses.

Let L_1 and L_2 (Fig. 169) be the lenses forming a doublet of a

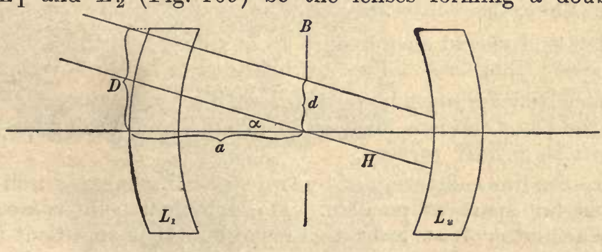

Fig. 169

diameter $2D$, let B be the plane of the diaphragm, and let the dia-

phragm be situated midway between L_1 and L_2. Let a be the distance of the diaphragm from the apex of the anterior surface, and let $2d$ be the diameter of the aperture of the diaphragm.

If, for the sake of simplification, the collective effect of the parts of the doublet be neglected, *i. e.*, if it be assumed that the diameter of the pencil passing through the diaphragm $2d$ is also $2d$ previous to the passage through the lens, and also if we disregard the curvature of the external surfaces and the thickness of the lenses, then the oblique pencil passing through $2d$ is stopped at that particular moment when the principal ray, H, is of that degree of obliquity which is represented by a straight line contained in a plane passing through the axis of the objective, and connecting the edge of the lens with the edge of the diaphragm on the same side of the axis. Let the angle between this principal ray, H, and the axis be α, then it will easily be seen that

$$\tan. \alpha = \frac{D-d}{a}$$

This limit is increased in a measure as the difference $D-d$ increases, *i. e.*, in a measure as the aperture of the diaphragm d becomes less.

When $D-d$ is constant, then α increases in a measure as a decreases.

From this we infer: the shorter the distance of the diaphragm from the extreme apices of the lenses, the later is the moment of the stoppage of oblique pencils by the edge of the lenses.

107. Surveying cameras.—The number of instruments devised or proposed for photographic surveying is considerable. They are divided into three classes:—

1st. Ordinary cameras.
2nd. Surveying cameras or "photogrammeters."
3rd. Photo-theodolites.

The first and second ones require an auxiliary instrument for measuring angles; photo-theodolites are intended to be employed alone.

Ordinary cameras must be provided with a level; the relative positions of the plate and lens must be invariable, and when adjusted, the plate must be exactly vertical.

The horizon line is determined by two zenith distances of well-defined objects as far apart as possible. The principal point is ascertained from the azimuths of at least three points. It is expedient to make these determinations for every photograph.

The employment of ordinary cameras for surveying cannot be recommended; satisfactory results cannot be expected from imperfect instruments.

There are many patterns of surveying cameras or photogrammeters. One of the earliest is Meydenbauer's. It is a camera with tapering bellows set on a horizontal circle; it moves on a vertical axis. A clamp and tangent screw serve to bring the optical axis in any desired azimuth Two metal rods connect rigidly the object glass and the frame bearing the plate. The levelling screws are part of the head of the tripod, the horizontal circle resting on the top of the screws. This arrangement dispenses with the usual triangular base of instruments and reduces the height considerably.

Finsterwalder's photogrammeter is, in external appearance, somewhat like Meydenbauer's, but the horizontal circle is set on the ordinary triangular base with the three levelling screws.

Meydenbauer has devised also a small camera which is made in two sizes, 6 × 8 and 9 × 12 centimetres. It is a magazine camera; the plates are changed through a bag placed underneath. The camera is supported in a peculiar manner; it is placed on the top of a vertical rod, which is jointed to the head of a tripod. The free ends of the tripod and the upper end of the vertical rod are connected by wires to which tension is given by ratchet wheels and pawls.

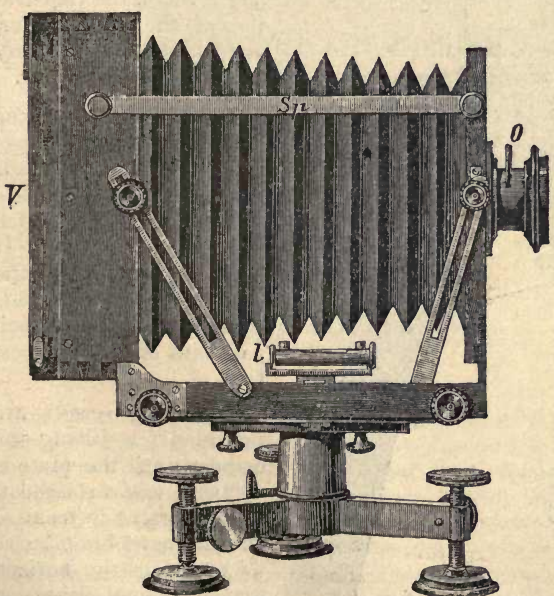

Fig. 170.

R. Lechner, of Vienna, Austria, makes the three following patterns.

Werner's (Fig. 170), is an ordinary folding camera set on a triangular base with levelling screws.

Pollack's photogrammeter (Fig. 171) is a rectangular metal camera mounted on a graduated circle with levelling screws. The rising front is moved by a rack and pinion, the displacement being read on a scale by a microscope and vernier. The object glass is an anastigmat $f/18$. In front of the ground glass is a metal frame graduated in centimetres. There is a special contrivance for pressing the plate in the dark slide against this frame, so that the centimetre scale is impressed on the photograph. This serves, firstly, to determine the vertical and horizontal lines, secondly, to eliminate any error in the registration of the plate holder, and, thirdly, to mark any distortion of the paper.

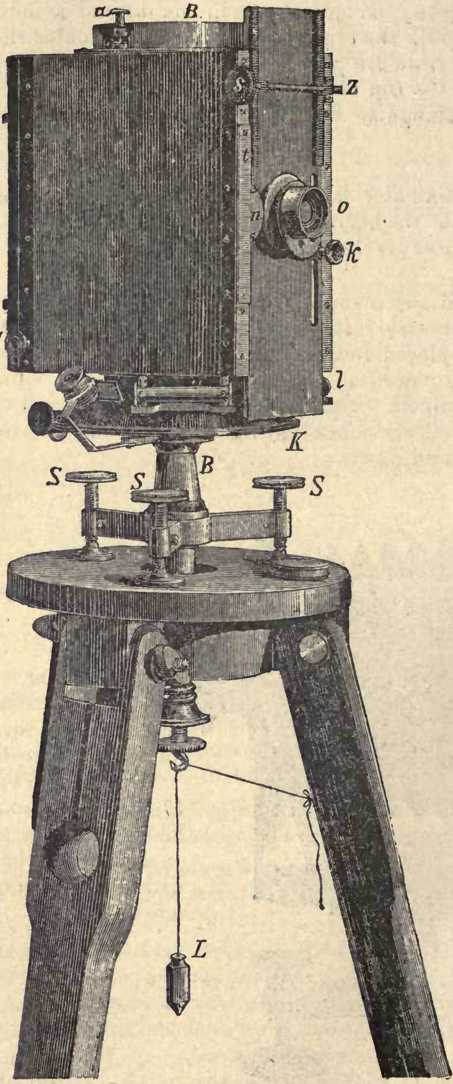

Fig. 171.

In the middle of the focussing screen is an eye piece with cross threads exactly opposite the object glass, with which it forms a telescope. It is so adjusted that the intersection of the threads coincides with the principal point of the perspective.

Adjusting screws are provided for tilting the camera until the plate is vertical, and for adjusting the frame in front of the photographic plate so as to bring the horizon and principal lines in correct position.

Huebl's "Plane table photogrammeter," by the same maker (Fig. 172), is a compact and very well conceived instrument. It has a camera base or carrier, on which are the levelling screws. It is put on the tripod and the camera is connected with it by a central screw with spiral spring. The tops of the levelling screws fit into slits at the bottom of the camera.

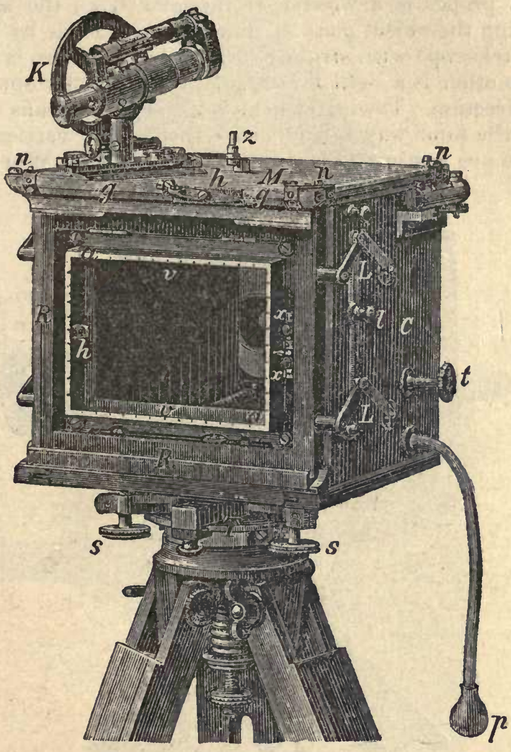

Fig. 172.

The back frame is, like Pollack's, graduated in centimetres, and the plate is brought in contact with it by a forward motion of the slide carrying the plate holder. The peculiarity of the instrument is the top, which is disposed for use as a plane table. A sheet of drawing paper is fixed to it, and the directions of important points are drawn by means of an alidade with telescope and vertical circle. When the alidade is employed, no additional instrument is needed, the azimuths being registered on the plane table and the zenith distances read off the vertical circle and noted.

Passing to photo-theodolites, we find among them the first instrument specially constructed for surveying purposes, Col. Laussedat's photo-theodolite, made by Brunner in 1858-59.

Fig. 173 shows this instrument as now constructed by E. Ducretet and L. Lejeune. The base of the camera is the lower part of a theodolite, with levelling screws, graduated horizontal circle and vernier. The camera proper is a wooden rectangular box; the sliding front board carrying the object glass is moved up or down by a rack and pinion. A telescope with striding level and vertical arc is fixed to one side; on the other is a sight for rapidly bringing the optical axis in any given direction. The instrument is levelled by means of the striding level in the same way as any other theodolite; vertical and horizontal angles are measured also in the same manner. When the line of

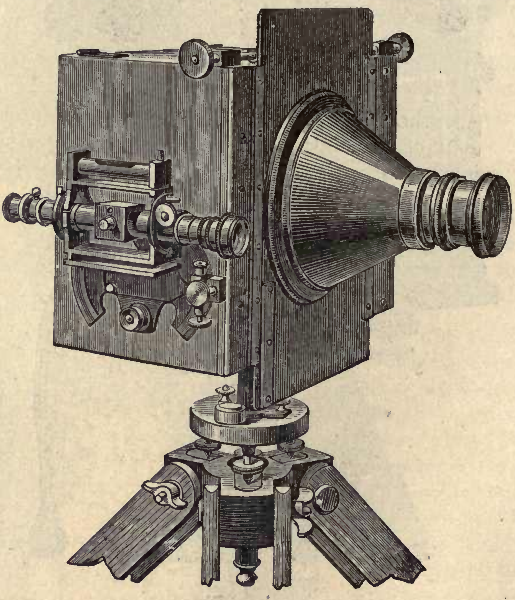

Fig. 173.

collimation of the telescope is horizontal, it must be directed on the principal point of the perspective. The registering marks of the horizon and principal lines are checked by bringing, with the striding level, the telescope to this position. It may occasionally be desired to use the instrument for measuring angles only without taking photographs, as, for instance, in making a subsidiary triangulation. It would be inconvenient to carry the camera when it is not needed. In this case, it is detached and the telescope set up as in Fig. 174; reduced to this form, it is an ordinary surveying transit.

Inversely, the telescope and vertical arc may be detached ; it then becomes a surveying camera.

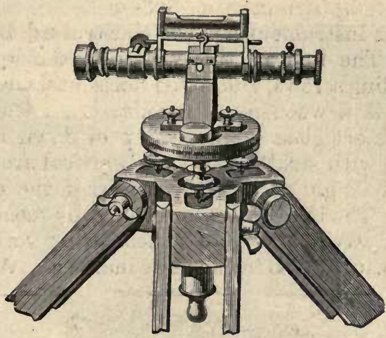

Fig. 174.

The metal parts of the instrument are made of aluminium ; it is compact, light and well executed. It answers well the purpose intended, which is to make detailed topographical surveys without having recourse

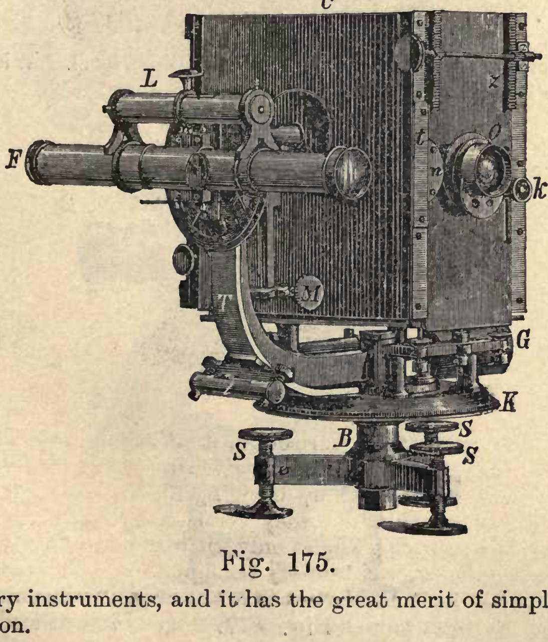

Fig. 175.

to auxiliary instruments, and it has the great merit of simplicity in its construction.

R. Lechner makes Pollack's photo-theodolite (Figs. 175, 176 and 177). The camera is the same as in the photogrammeter, and there is

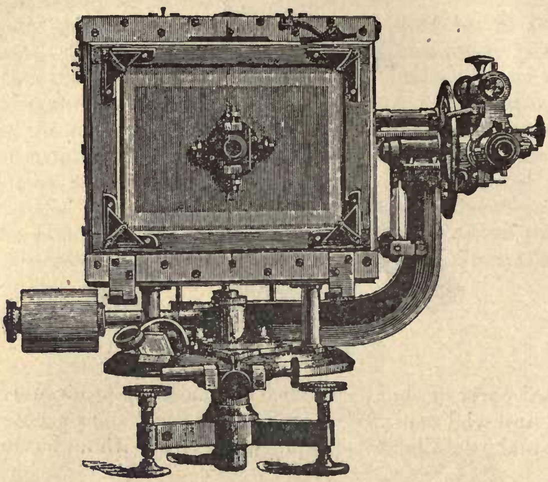

Fig. 176.

added a vertical circle, telescope and striding level. Without the camera, the instrument is an ordinary theodolite.

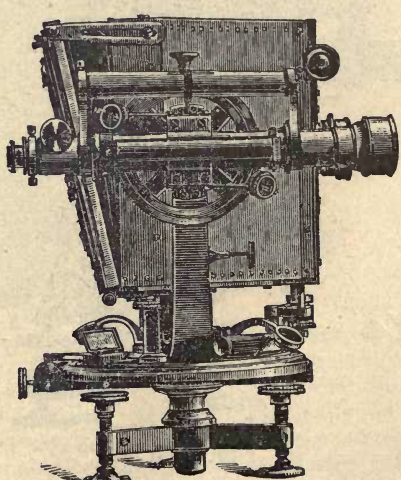

Fig. 177.

The photo-theodolite of the Geographical Military Institute of Italy is shown in Figs. 178 and 179. The camera is a pyramidal metal box moving on a horizontal axis, so that it may be inclined to the horizon when the angle of view in the vertical position is not sufficient to take in all the points of the landscape.

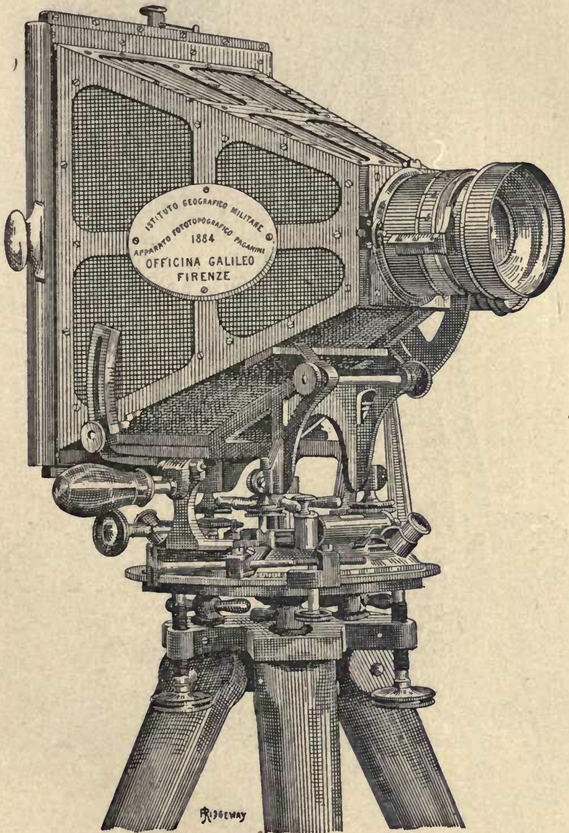

Fig. 178

This arrangement exists also in Koppe's theodolite, and may be useful in a very broken country. Generally a vertical field of 45° is quite

sufficient, and most of the instruments are made for vertical views only.

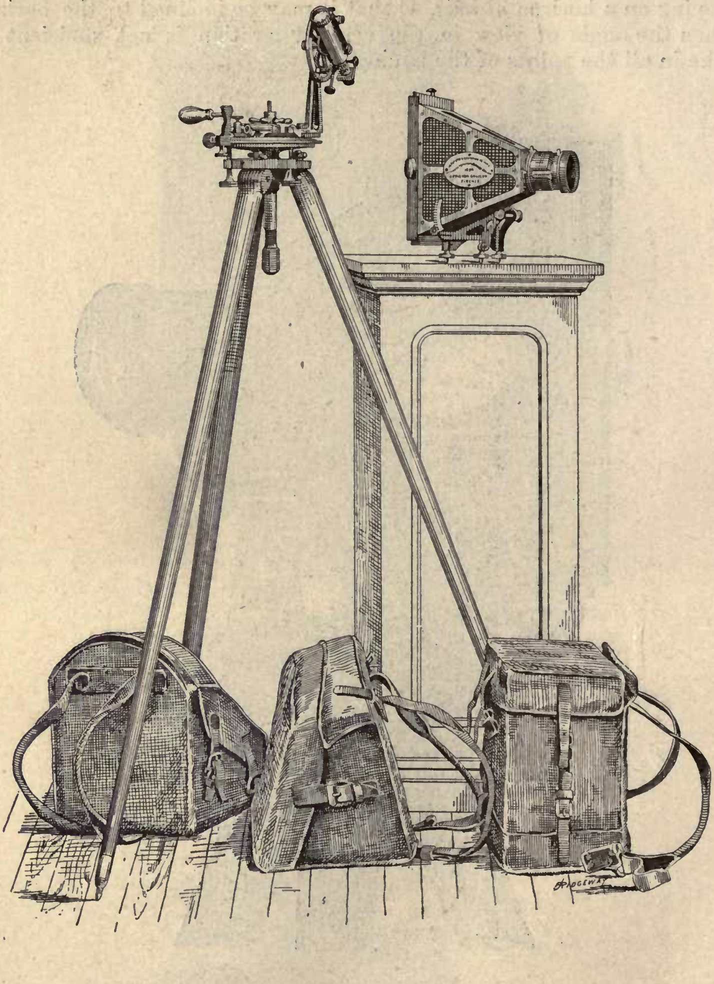

Fig. 179

The levelling screws are part of the tripod head, as in Meydenbauer's photogrammeter, and they support the horizontal circle. The vertical circle, telescope and level are on an upright at the side. The object glass moves in its tube so that it can be adjusted to the focus

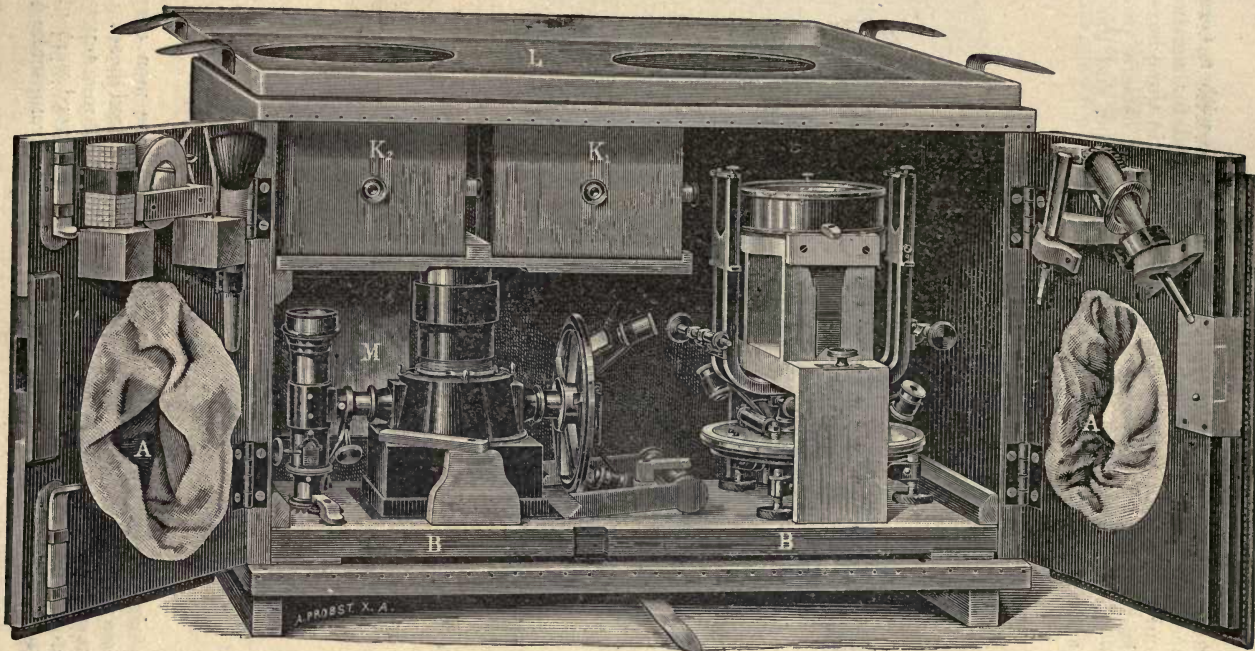

Fig. 180.

of near objects. The displacement is read on a scale with a vernier. The camera can be detached and the instrument used for measuring angles only, as an ordinary theodolite. It is packed in three boxes of convenient shape for transport ; one contains the camera proper, another one the two circles, tripod head and telescope, and the last one, the plate holders.

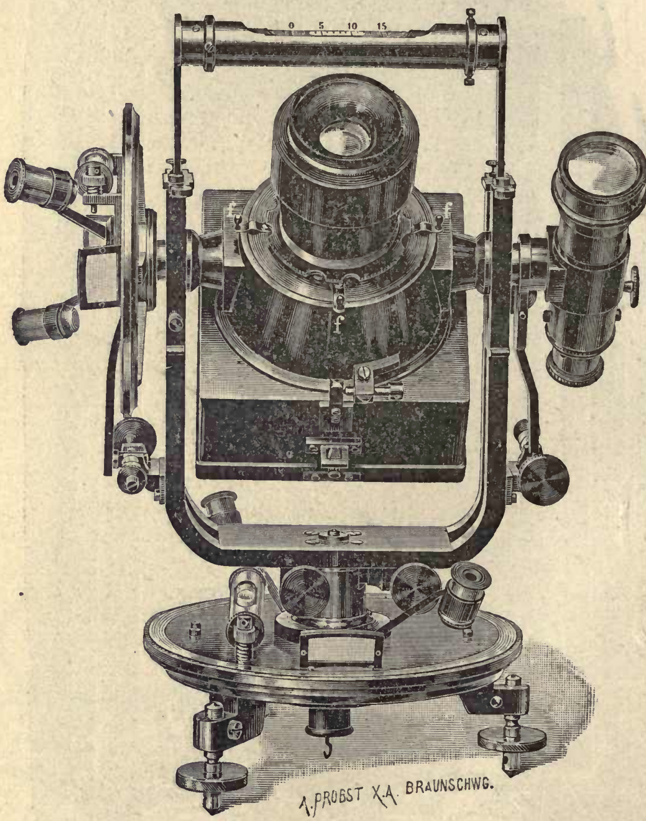

Fig. 181.

The tripod is formed of the alpenstocks of the three men carrying the cases ; they are fixed with thumb-screws to the tripod head. This instrument was constructed for use in the Alps, where men must have alpenstocks and where the tripod is the most awkward part of the engineer's equipment. The alpenstocks being very strong make a firm

stand for the theodolite, and the transportation of an ordinary tripod is dispensed with.

Koppe's photo-theodolite (Figs. 180, 181 and 182) is an ordinary theodolite with telescope on one side and vertical circle on the other. In the middle of the horizontal axis, and fixed to it, is the camera. The

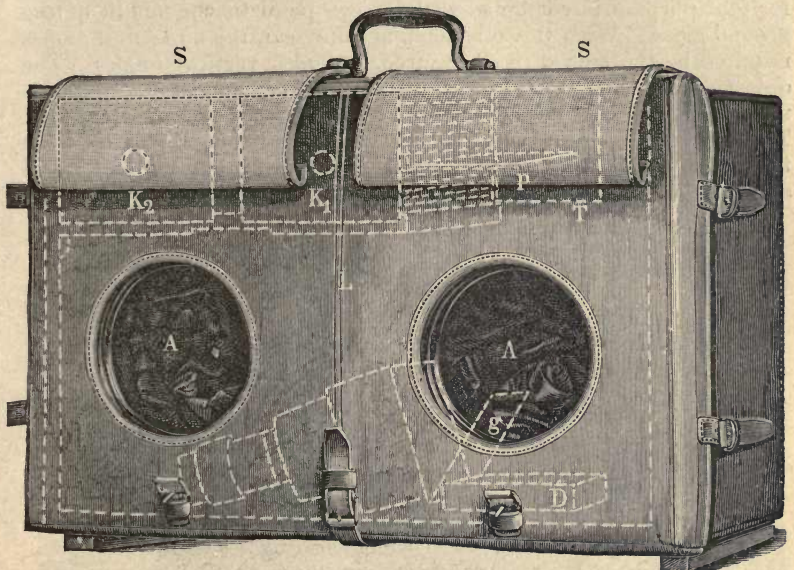

Fig. 182

difference in the Italian instrument is that the telescope moves with the camera, their axes remaining parallel. There are no dark slides; the plates are changed in the packing case, where they are stored in two boxes K_1 K_2, Fig. 180.

In the lids are two circular holes AA, to which are attached sleeves of flexible material impervious to light. For the purpose of changing a plate, the case is closed after placing the camera in it; the hands are inserted through the sleeves, the exposed plate is taken out and pushed into box K_1; a new plate is then taken from box K_2, put in position, and the camera closed.

It is evident from the descriptions given that some instruments are intended only for surveys plotted by geometrical constructions, while others are designed to furnish results of the utmost precision, the measurements on the photographs being made with special appliances and submitted to calculation. The photo-theodolite of

Starke & Kammerer, of Vienna, is an instrument representative of the latter class, and will be described with a little more detail; the remarks apply to other instruments of the same kind.* The general principle of these instruments is to have a telescope moving in a plane parallel to the principal plane of the perspective; in this one, the object glass of the camera takes the place of the object glass of the telescope, and for that purpose is combined with an eye piece in the middle of the ground glass. With this telescope, not only can the horizontal angles required by the survey be measured, but also vertical angles. The object, however, is not to make extensive series of angular measurements, but only to take readings on the points of the triangulation. The three levelling screws of the base rest in slits on the head of the tripod to which the base is fastened by a central screw and spiral spring. In Fig. 183, the levelling screws appear in A, A, A. The base

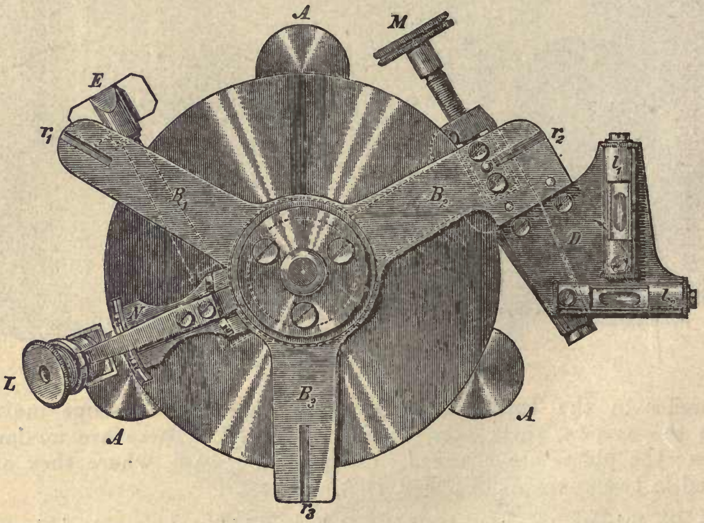

Fig. 183

carries the vertical axis and horizontal circle; the latter is graduated to 20′ on its vertical face, and is read to 1′ by means of a vernier N and microscope L. The end of the vertical axis carries three arms, B_1, B_2, B_3, to which the vernier is screwed. The vertical axis is adjusted by means of the three levelling screws and of the two levels l_1, l_2. The plate, on which are the levels, is screwed to the arm B_2. A clamp E and tangent screw M complete the base.

*This description is from a paper by G. Starke in the "Zeitschrift des Oesterreischischen Ingenieur und Architekten Vereines," No. 5 of 1894.

In the slits r_1 r_2 r_3 of the arms B_1 B_2 B_3 rest the three levelling screws F_1 F_2 F_3 of the camera (Fig. 184); the camera is screwed from

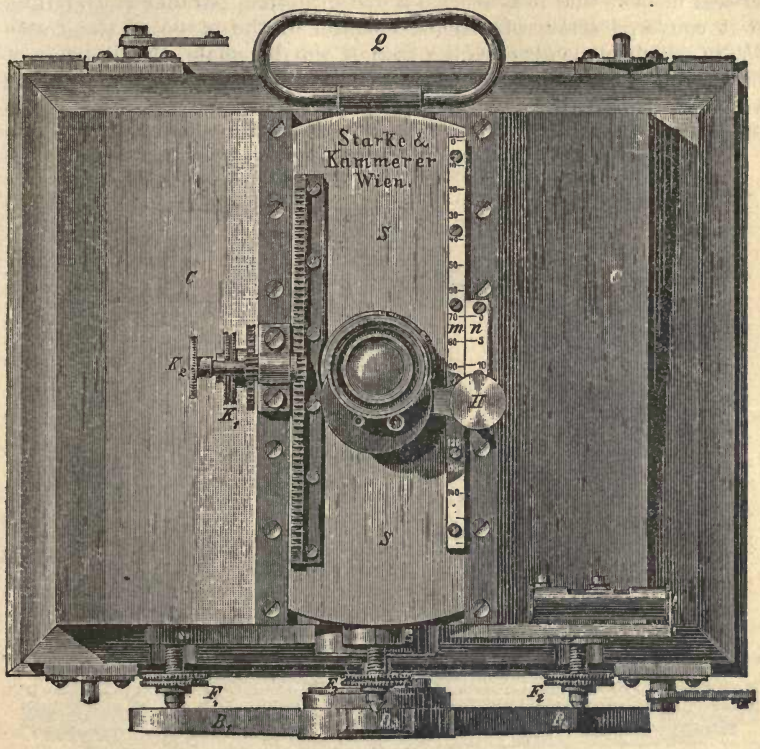

Fig. 184.

the inside to the lower part. A handle, Q, is on top for convenience in lifting it. The camera has, as well as the lower part, two cross levels l_3 l_4 (Fig. 185), for adjustment with the foot screws F_1 F_2 F_3. Fig. 184 shows the camera from the lens side, and Fig. 185 from above. Fig. 186 is the frame securely bound to the camera which fixes the position of the focal plane; the ground glass and the dry plates are brought in contact with it. It consists of four scales at right angles with notches at every centimetre. Those in $a_1 a_2$ and $b_1 b_2$ are the middle marks; when joined, the intersection is the middle of the picture. The size of the picture is $17{\cdot}8 \times 22{\cdot}8$ centimetres. By construction, the notches are so placed that $a_1 a_2$ is exactly perpendicular to $b_1 b_2$; thus when $a_1 a_2$ is horizontal, $b_1 b_2$ is in the vertical plane perpendicular to $a_1 a_2$.

The lens is a Zeiss anastigmat, $f/18$ of about 212 millimetres focus. It is screwed on the plate S, Figs. 184 and 185, which has a sliding vertical motion, and it is at such a distance that parallel rays falling on it converge after refraction to points in the plane of the frame RR, in which plane also are the face of the dry plate, the dull side of the ground glass and the threads of the diaphragm in front of the eye piece. The distance of objects photographed for surveying is generally so great that a fixed focus has no disadvantage, and the same

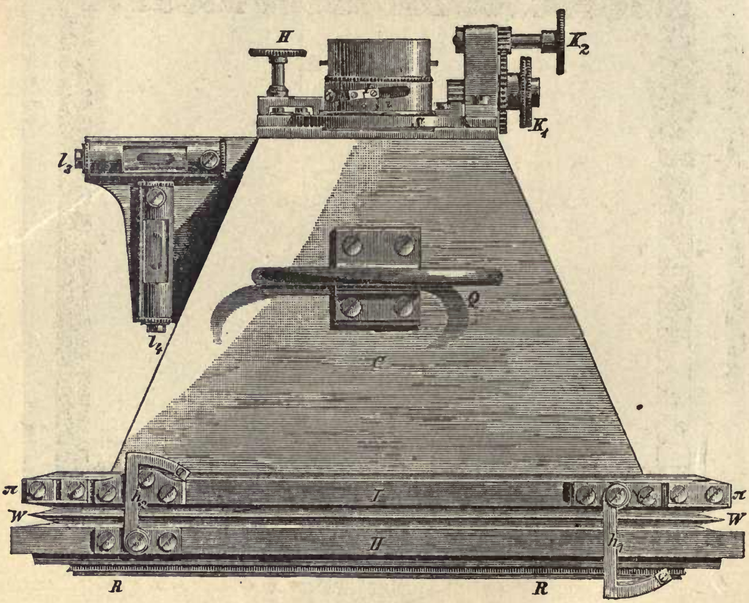

Fig. 185.

focal length may be employed throughout. For distances of 500, 400, 300, 200 and 100 metres and with a lens of 212 millimetres focus, the displacement of the focal plane should only be 0·09, 0·11, 0·15, 0·22 and 0·45 millimetres respectively. For shorter distances, the lens has a motion in the direction of the axis to the extent of two millimetres. This brings in focus objects situated at 23 metres from the instrument. In the helicoidal slit cut in the outer tube of the lens is a small block t screwed to the inner tube. Both tubes are clamped together by the screw shown at the end of the slit. Unscrewing it and turning the inner tube from right to left brings the lens out. The nclination of the slit to the axis is such, as to correspond to a motion

of two millimetres in the direction of the axis when the block t travels from one end of the slit to the other. These two extreme positions are indicated by the graduations 0 and 2 on the outer tube opposite the index mark on t. The interval is divided into twenty parts, so that the displacement of the lens is read directly to 0·1 millimetre. This graduation is, however, seldom used; in the generality of cases the index is set at zero and the lens clamped in that position.

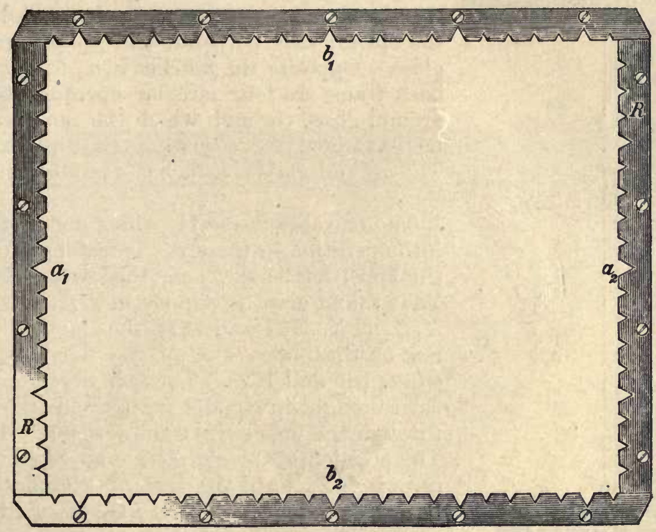

Fig. 186.

A vertical motion is given to the lens by the rack and pinions $K_1 K_2$ Figs. 184 and 185; the latter, K_2, is for slow motion, H is the clamp. The displacement is measured on a scale m divided to millimetres with the vernier n reading to 0·05 millimetre. This scale is 140 millimetres long; from 0 to 70 the optical axis is directed below the horizon, and from 70 to 140 is directed above. At 70 it should be exactly on the horizon.

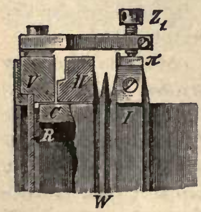

Fig. 187.

Two frames I and II (Figs. 187 and 188), serve to make a light, tight connection between the single plate holders or ground glass and the camera. They are connected by the bellows W; I is fixed to the camera, but II can move as far as the bellows extend. Each frame carries two hooks; the frame I has the upper right one, h_1, and an exactly similar

one, lower left. The frame II has the upper left hook h_2 and a lower right one. These hooks fasten the ground glass and plate-holders to the camera. The ground glass is shown in Fig. 189; in Fig. 187, it is represented in section attached to the camera. The outer wooden frame carries two metal diagonals connected to a central ring. In the middle of the ring is the eye piece movable on the axis $x_1 x_2$, but which may be fastened by the two flaps $p_1 p_2$ in its normal position with the optical axis perpendicular to the ground glass. Opposite the notches $a_1 a_2$ $b_1 b_2$ of the back frame are four circular openings in the ground glass, through which the notches may be examined for adjusting the instrument. The ground glass is attached as follows :—

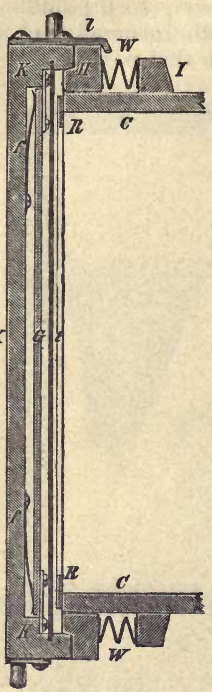

Fig. 188.

The movable frame II, which does not come into operation in this case, is fastened to I by the upper left hook h_2 and the lower right one. The ground glass is supported by the screws $Z_1 Z_2$ (Figs. 187 and 189), the points of which rest on the plates π π' of the fixed frame I (Figs. 185 and 187). The face of the glass is then brought in contact with the back frame through the upper right and lower left hooks. The position of the eye piece is adjusted by the screws $Z_1 Z_2$ until the line of collimation is horizontal when the object glass is in its normal position at 70 of the vertical scale. In this position, or not far from it, points can be sighted through the eye piece of the ground glass, but when the lens is further away from the normal position, the use of the eye piece with its axis perpendicular to the ground glass becomes inconvenient or impossible. In such cases the flaps $p_1 p_2$ are unfastened, and the eye piece is rotated on the axis $x_1 x_2$ until its optical axis is in the direction of the object glass. No great precision is required; a few trials are sufficient to find the inclination for which the best image is seen.

The plate holder K, shown in section, Fig. 188, contains the dry plate G which is pressed by springs at the four corners. The hard rubber shutter t is completely withdrawn during exposure. The adjustment of the plate holder to the camera takes place as follows :— Frame II is set free from I, and the holder hung on frame II by the bent plate l: the projecting edge of the holder engages in the rebate of frame II and makes a light, tight joint. It is then fastened to frame II by the pair of hooks upper left and lower right. Fig. 188

shows this position of the holder. Having made sure that the cap is on the lens, the shutter can now be withdrawn, after which the pair of hooks upper right and lower left are brought into play and draw the holder forward until the dry plate is brought in contact with the back frame *R R* of the camera. The exposure is then given by uncapping the lens and the holder withdrawn by repeating the same operations in inverse order, first unfastening the pair of hooks upper right and lower left, inserting the shutter and drawing back the two last hooks, upper left and lower right.

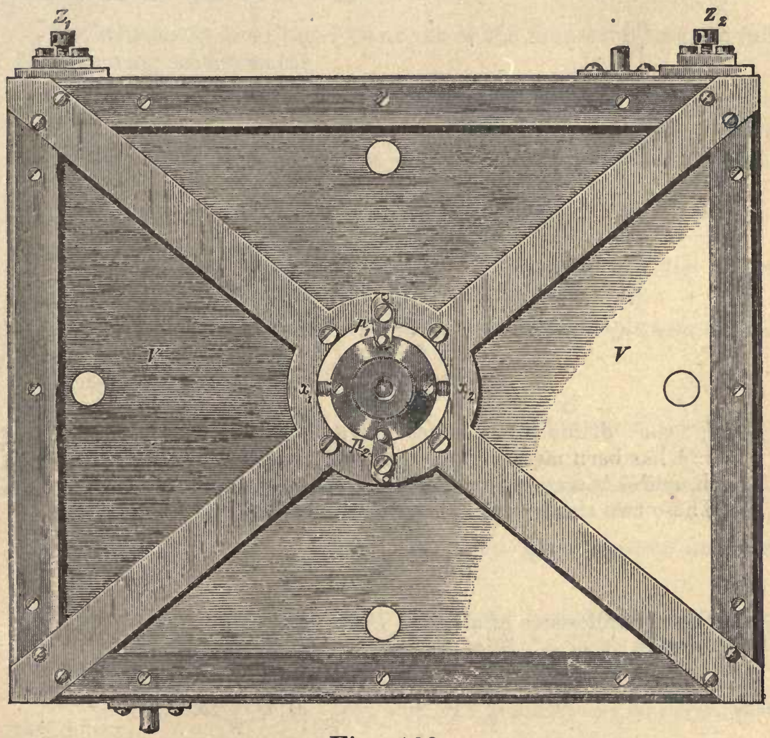

Fig. 189.

If the stations are points of the triangulation or have previously been determined, the only angles to be measured are those required for the orientation of the views: a vertical arc or circle is unnecessary. But it may happen that the station has not been fixed or could not be seen from the summits of the triangulation; in such cases, vertical angles must be measured to obtain the altitude of the photographic

station. Let O, Fig. 190, be the second nodal point of the lens, C the intersection of the diaphragm threads; the instrument being adjusted, CO is horizontal when the lens is at 70 of the vertical scale. Let M be a known point from which the position of the photographic station must be ascertained; HM is the altitude to be found. The eye piece is turned in the direction CM, and the object glass raised until the image of M is seen at the intersection of the threads of the diaphragm; the object glass plate is clamped and its height read on the vertical scale. Let V be the intersection of the vertical axis of the instrument with the horizontal axis of the camera. In the triangles MHC,

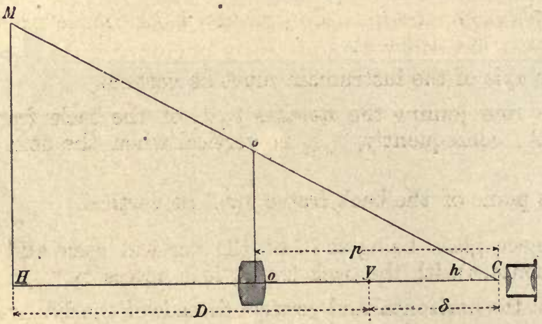

Fig. 190

$O'OC$, the distance D must have been previously ascertained, $OO' = h$ has been measured on the vertical scale, $OC = p$ is the focal length, and δ is a constant of the instrument which can be scaled on it. These two similar triangles give the proportion

$$\frac{H}{h} = \frac{D+\delta}{p}$$

H being the difference of altitude HM.

Therefore

$$H = \frac{h}{p}(D+\delta) = \frac{h}{p}D + \frac{h}{p}\delta.$$

For this instrument $\delta = 100$, $p = 212$ mm. and the maximum value of h is 70 mm. Then $\frac{h}{p}\delta$ cannot exceed 0.33 mm., a quantity which can always be neglected, the expression of H taking the simpler form of

$$H = \frac{h}{p}D.$$

Let $\triangle H$ be the error of H due to an error of $\triangle h$ in the reading of h:

$$\triangle H = \frac{D}{p} \triangle h$$

h is read on the scale to 0·05 mm., so

$$\triangle H = \pm 0{\cdot}000118\, D.$$

This gives 120 mm. for $D = 1000$ metres.

The instrument is adjusted by means of the four levels so as to fulfil the following requirements:

1. The axis of the instrument must be vertical.

2. The line joining the notches $a_1 a_2$ of the back frame must be horizontal: consequently, $b_1 b_2$ is vertical when the next condition is fulfilled.

3. The plane of the back frame must be vertical.

The object glass being at 70 of the vertical scale and the ground glass in contact with the back frame, it is necessary:

4. That the horizontal and vertical threads of the diaphragm coincide with the lines $a_1 a_2$, $b_1 b_2$, of the back frame.

5. That the line of collimation be horizontal.

6. That the threads of the diaphragm lie in the plane of the dull side of the ground glass.

7. That the horizontal thread of the diaphragm coincides with the axis of rotation $x_1 x_2$ of the eye piece.

8. That the vertical motion of the object glass shall be on a line parallel to the line $b_1 b_2$ of the back frame.

Some of these adjustments must have been made by the instrument maker, and if incorrect can only be rectified by him. Those which follow are made by the engineer.

1.—The cross levels of the base are adjusted as in any other theodolite.

2 and 3—Cross levels of the camera.—Find a distant point of which the image comes in the apex of the notch a_1, and verify the coincidence with a magnifying glass through the circular hole in the ground glass. Then revolve the instrument on its vertical axis until the image comes into the notch a_2. Should it coincide exactly with the apex, then the line $a_1 a_2$ is horizontal; if not, it must be adjusted by the levelling screw under a_2. A few trials will soon bring it to the

right position ; the level parallel to a_1a_2 can now be adjusted and the bubble brought between its marks.

The verticality of the plane of the back frame is checked by a light plumb bob suspended from one of the hooks. It is adjusted by means of the foot screw F_3 under the object glass. When exactly vertical, the level l_4 is adjusted until the bubble comes within the marks. It is clear now that a_1a_2 will be horizontal and b_1b_2 vertical whenever the camera is levelled, so long as the adjustment of the cross levels does not change.

4. Selecting a distant point very close to the horizon, its image is brought into the apex of one of the notches a_1 or a_2 and brought to exact coincidence by a vertical motion of the object glass. The instrument is then revolved on its vertical axis until the image is seen in the eye piece. Should it coincide with the horizontal thread of the diaphragm, then the latter is in proper adjustment ; if not, it must be brought to coincide by raising or lowering the ground glass by means of the screws Z_1Z_2, half of the error being corrected by one screw and half by the other. They must both be screwed or unscrewed the same number of turns.

The vertical thread of the diaphragm must coincide with the line b_1b_2. To verify this, select a distant point and bring its image, by a slow motion of the horizontal circle, into exact coincidence with the edge of the back frame above or below the notch a_1 and read the angle on the horizontal circle. Revolve the instrument, and bring in the same way, the image of the point in contact with the edge of the frame above or below the notch a_2 ; read again the angle on the horizontal circle. Set the vernier in the middle of the measured angle and the image must coincide with the vertical thread of the diaphragm. The error, if any, is corrected by again using the screws Z_1Z_2. Should for instance, the thread be on the left of the image, Z_1 is unscrewed till half of the error is corrected and Z_2 screwed in to correct the other half. Inversely, Z_1 must be screwed in and Z_2 unscrewed when the thread is on the right of the image. It is clear that this correction will somewhat disturb the preceding one and that both will have to be repeated several times.

5. The line of collimation is adjusted to horizontality by the usual methods of the level instrument. The object glass may then not be exactly at 70 of the vertical scale. The index error may be employed to correct all the readings, or it may be eliminated by loosening the two screws of the vernier and moving it vertically until the reading is exactly 70.

The vertical motion of the object glass is verified by setting it at about 130, selecting a distant point and bringing its image into exact coincidence with the apex of the notch b_1, then lowering the object

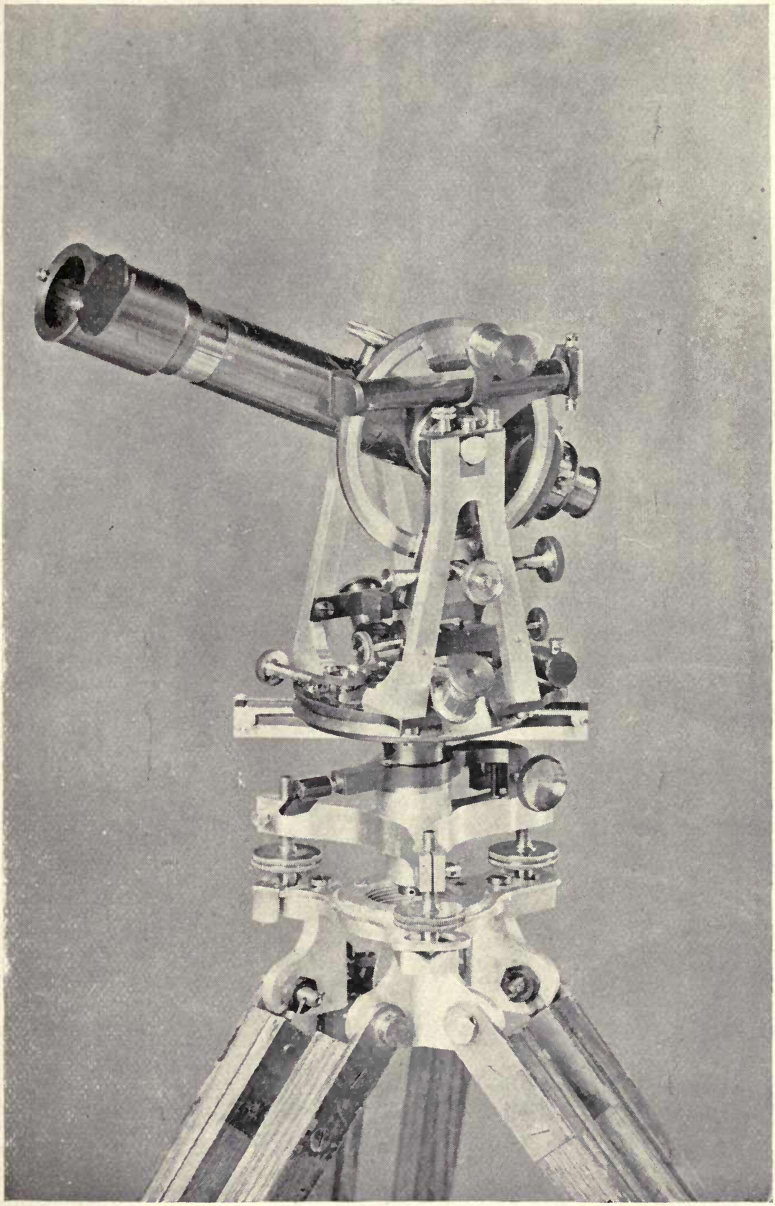

Fig. 191—TRANSIT THEODOLITE.

Fig. 192–CAMERA
of the Canadian Surveys (horizontal position).

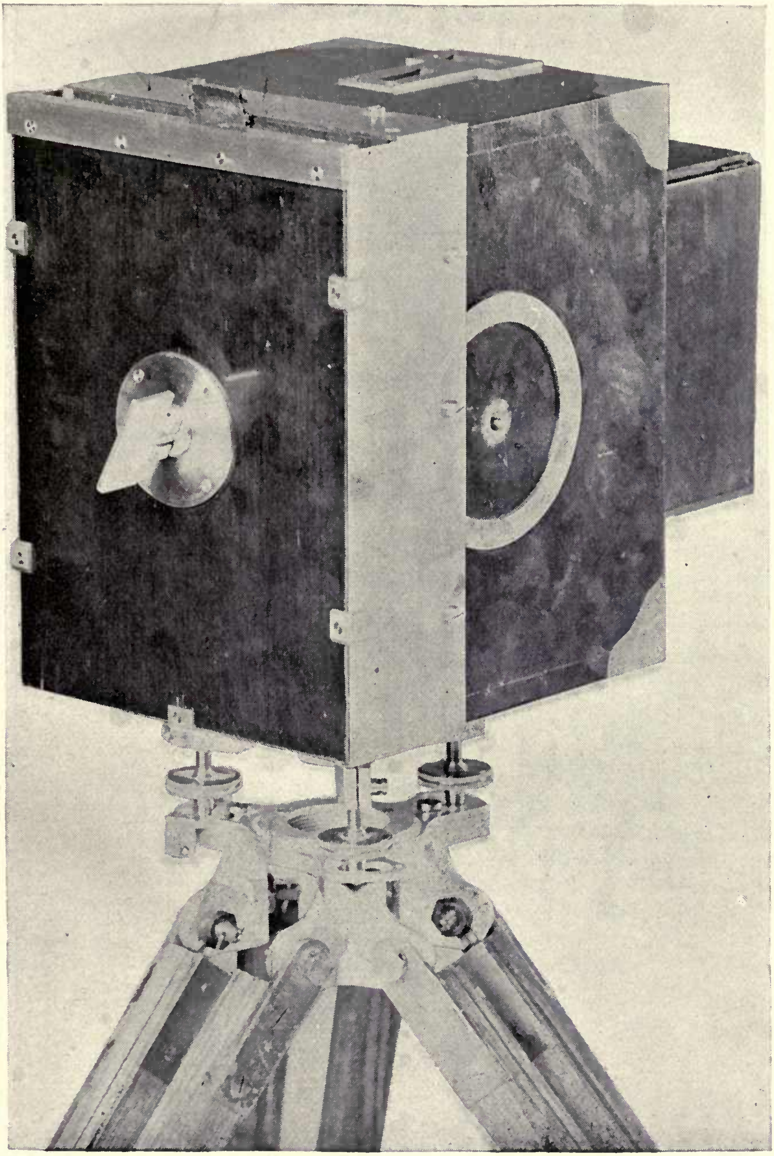

Fig. 193—CAMERA
of the Canadian Surveys (vertical position).

glass till the image comes into the notch b_2, where it must coincide with the apex. The image must also be seen on the vertical thread of the diaphragm when the object glass is at the proper height. These conditions being fulfilled, the displacement of the object glass is on a line parallel to $b_1 b_2$ and therefore vertical, and the vertical thread coincides with the line $b_1 b_2$.

108. Canadian equipment.—The equipment of a party on the Canadian Surveys consists of a transit theodolite and two cameras. The transit theodolite and its tripod are carried by the surveyor, and a camera without the tripod by one of the men who always accompanies the surveyor. The assistant has his own camera with a tripod.

The transit is one of the ordinary patterns used by surveyors and is shown in Fig. 191. It has three inch circles and reads to minutes. The tripod is a short one, specially designed for mountain work. It is three feet four inches long and has sliding legs, the joint being perfectly stiff. The surveyor observes either in a sitting or kneeling position. For the purpose of packing, the head of the tripod is taken off and put in the transit box; when folded, the legs are twenty inches long and are placed under the box of the transit as shown in frontispiece. The heavy parts of the instrument are made of aluminium; the whole, including tripod and case, and also the camera base, weighs fourteen pounds and eight ounces.

The camera is shown in Figs. 192 and 193; Fig. 194 represents sections of the instrument. The camera proper is a rectangular metal box *AB*, open at one end. It carries the lens *L* and two sets of cross levels *CC*, which are read through openings in the outer mahogany box. The metal box is supported by wooden blocks and by a frame *FF* held in position by two bolts *DD*. The plate holder is made for single plates; it is inserted in a carrier *EE*, which can be moved forwards and backwards by means of the screw *G*. A folding shade *HH* hooked in front of the camera, and diaphragms *KK* inside of the metal box, intercept all light which does not contribute to the formation of the image on the photographic plate. The camera rests on a triangular base with foot screws, identical with the base of the transit, so that both may fit on the same tripod. It may be set up with the longer side either horizontal or vertical.

The lens is a Zeiss anastigmat, No. 3 of series V, 141 millimetres focus, with a deep orange screen in front.

Having set up the camera on the tripod, the plate holder carrier *E* is moved back as far as it will go by turning the screw *G*; the plate holder is inserted through the opening *M*, the slide is withdrawn, and the carrier moved forward by the screw *G* until the plate is in contact

with the back of the metal box. In order to secure perfect contact, the carrier has a certain amount of free motion. The camera must

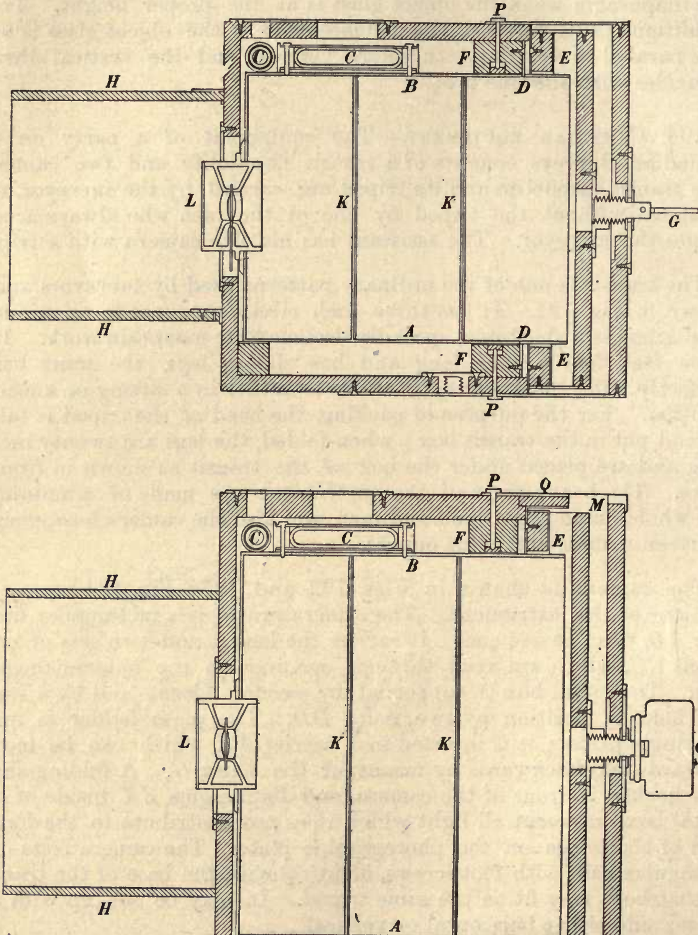

Fig. 194

now be turned in the proper direction; the field embraced by the plate is indicated by lines drawn on the outside of the mahogany box. The

camera is then carefully levelled, the exposure given, and the plate holder withdrawn by repeating the same operations in inverse order.

The levels are rigidly fixed to the camera without any means of adjustment. They are, however, very nearly adjusted by the maker. For this purpose, he takes out the metal box and places it on a piece of plate glass which has been levelled like an artificial horizon. By filing one end or the other of the levels' outer case, he brings each bubble very nearly in the middle of its tube. These tubes bear continuous graduations as illustrated in Fig. 195.

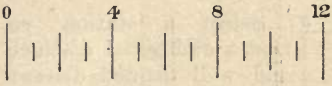

Fig. 195

Accompanying the camera is a piece of plate glass, $\frac{1}{4}$ in. thick and 11 in. long, which can be inserted in the carrier instead of the plate holder. The end of the glass, which projects outside of the camera, is coated on the back with a varnish of gum guaiacum dissolved in alcohol to which some lamp black has been added. This varnish has very nearly the same refractive index as glass, and stops all reflections from the back of the plate glass.

The first thing to be done when the camera is received from the maker is to ascertain the exact readings of the levels when the back of the metal box, on which the photographic plate is pressed, is vertical. To do this, the bolts P (Fig. 194), next to the opening M, are unscrewed and removed: Q may then slide backwards and be taken out. The piece of plate glass is now inserted in the carrier E, and pressed in contact with the metal box. The camera is placed on its tripod and levelled. Immediately in front and at the same height, a transit T, Fig. 196, is set up, and after carefully adjusting it, a distant point P is selected on the same level as the transit and camera. The intersection of the threads of the telescope is brought to coincide with P, and the telescope is clamped to the vertical circle. Turning it around the vertical axis, the image of P reflected by the plate glass should appear upon the intersection of the telescope's threads. If it does, the face of the plate glass is vertical and the position of the bubble in the level tube is the correct one for adjusting the instrument. If it does not, the camera must be inclined forward or backward by means of the foot screws until coincidence is established. The bubble of the level may or may not be now in the middle of the

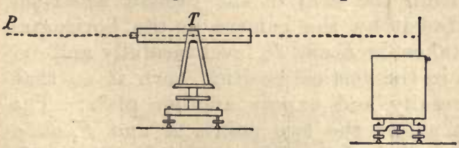

Fig. 196

tube, but its position, whatever it is, is the correct one for adjusting the camera. The divisions of the graduation between which the bubble is comprised are therefore noted, and whenever the camera is to be levelled, it must be remembered that the bubble is to be brought between these two same divisions.

This determination is made for the two positions of the camera, horizontal and vertical.

The next step is to fix the place of the principal point on the photographic plate, and to measure the distance line or focal length.

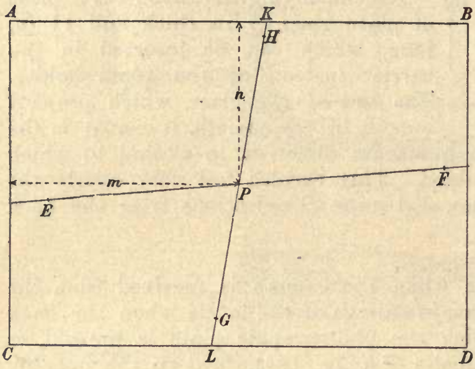

Fig. 197

Select a station so that a number of distinct and well defined distant points may be found on the horizon line. The view may be, for instance, the distant shore of a lake, or a large building, or rows of buildings. Set up the tripod and adjust the transit. Find two points E and F on the horizon line, or with a zenith distance of 90°, so that they both come within the field of the camera, when set horizontal, and near the edges of the plate. Measure the angle ω between them. Find two other points G and H, also on the horizon line, and such a distance apart that they both come within the field of the camera when set vertical. Now, replace the transit by the camera in the horizontal position, turn it so that it takes in E and F, level carefully and expose a plate. Set the camera in the vertical position, turn it so that it takes in G and H, level carefully and expose another plate. The first plate after development, shows the two points E and F, on a line very nearly parallel to the edges AB and CD, Fig. 197, of the metal box. The principal point is of course on this line. Cut the line through the film with a fine needle point.

The second plate, exposed in the vertical position, gives another horizon line GH which may be transferred to the first plate by means of the distances AK, CL to the corners of the metal box. This line is also cut through the film with a fine needle: the principal point P is at the intersection of both horizon lines.

The distance line is calculated from the angle ω between E and F and from the distances EP and PF.

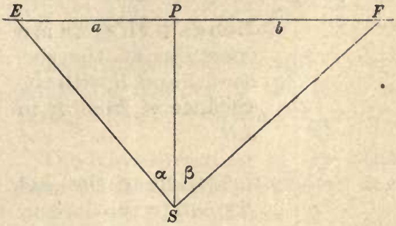

Fig. 198

Let S, Fig. 198, be the second nodal point of the lens ; a and b the distances EP and PF, α and β the angles between E and P, and between P and F.

$$\alpha + \beta = \omega$$

a and b are measured on the plate. Designating by f the focal length PS, we have

$$\tan. \alpha = \frac{a}{f}$$

$$\tan. \beta = \frac{b}{f}$$

$$\tan. \alpha \tan. \beta = \frac{ab}{f^2}$$

Hence :

$$\tan. (\alpha + \beta) = \tan. \omega = \frac{\frac{a}{f} + \frac{b}{f}}{1 - \frac{ab}{f^2}} \quad \text{or}$$

$$f^2 - \frac{a+b}{\tan. \omega} f - ab = 0 \quad \text{and}$$

$$f = \frac{a+b}{2 \tan. \omega} + \sqrt{\frac{(a+b)^2}{4 \tan.^2 \omega} + ab}$$

Having now found the focal length and principal point, reference marks have to be made on the edges of the metal box to indicate the horizon and principal lines and the focal length on the prints or enlargements from the negatives.

Measure the distance m (Fig. 197), from P to AC. From the corresponding corners A and C (Fig. 199), of the metal box, lay out m in AR and CT. With a very fine and sharp file, held in the direction

of the lens, cut in the edge of the metal a clean and sharp notch at T and another one at R.

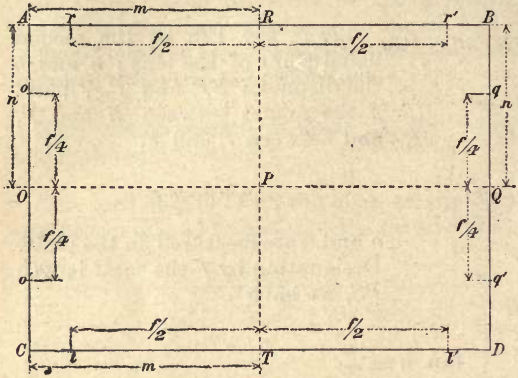

Fig. 199

Repeat the same operation at the corners A and B with the distance n from P to AB.

The lines OQ and RT will be the horizon and principal lines of the photographs when the camera is properly levelled.

From R and T, measure the distances Rr, Rr', Tt, Tt', equal to one half of the focal length. From O and Q measure Oo, Oo', Qq, Qq', equal to one quarter of the focal length, and at each one of these points make another notch with the file held in the direction of the lens. Every photograph will now show, like those which accompany the specimen plan, twelve triangular projections into the black border of the photograph. Four of these projections serve to fix the horizon and principal lines ; the remaining eight give the focal length.

It is now necessary to find the correct readings of the transverse levels, when the horizon and principal lines pass exactly through the notches of the metal box.

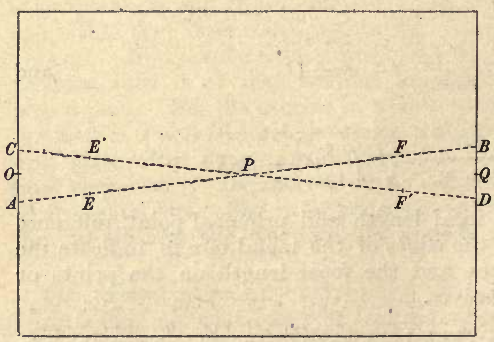

Fig. 200

Set up the camera again in front of the same distant view as before, but in adjusting it, bring the bubble of the transverse level near one end of the tube ; note the reading of the graduation and expose a plate. When developed, it will give an horizon line EF (Fig. 200), cutting the border of the negative in A and B, at some distance from the notches O and Q. Now change the adjust-

ment of the camera by bringing the bubble of the transverse level to the other end of the tube, note the reading of the level and expose another plate. This one gives another horizon line $E'F'$, cutting the border of the negative in C and D.

After measuring CO and OA or BQ and QD, a simple proportion gives the reading of the level which shall bring the horizon line through the two notches O and Q.

The correct reading of the other transverse level is found by the same method, with the camera in the vertical position.

All these operations must be executed with great care and precision, and with the help of a microscope of moderate power.

It has been assumed that the levels were placed very nearly in correct adjustment by the maker. If found too much out, they must, of course, first be approximately adjusted by setting the metal box on a well levelled plane. For this purpose, the plate glass supplied is set on the camera base and levelled like an artificial horizon.

109. Use of instruments.—The instruments and tripod being very light, steadiness is secured by a net between the tripod legs in which a heavy stone is placed. With this device, better photographs and observations are obtained, and there is no risk of the instrument being blown away during one of the wind blasts so frequent in the mountains.

After coming to a triangulation station, the surveyor adjusts his transit, and measures the azimuths and zenith distances of the triangulation points and of the camera stations. If accompanied by his assistant, each reads one vernier and enters the reading in his book. After completing the observations, they compare notes: any error is corrected on the spot.

The camera is carried in a leather case containing also twelve plate holders; when more holders are wanted, they are carried separately. Taking the camera out of the case, the levelling base is screwed to it and put on the tripod; the shade is unfolded and attached to the front. A plate holder is inserted in the carrier, and its number noted as well as the approximate direction of the view. Having made sure that the cap is on the lens, the surveyor draws the slide and screws the plate in contact with the metal box. He now turns the camera around until the lines on the upper face show that it is properly directed. He looks along the lines of the side face to see whether the view reaches high or low enough; if it does not, he puts the longer dimension of

the camera upright, unless already in that position. He levels carefully and exposes the plate. When the sun shines inside of the front hood, it should be shaded off by holding something above the hood. On no account must the sun be allowed to shine upon the lens.

The most important camera stations are occupied by the surveyor; the other stations by the assistant with his own camera.

All views are taken with the same stop, F/36.

CHAPTER V.

Hurter & Driffield's Investigations.

110. General remarks.—The laws which govern the behaviour of photographic plates under the action of light and developers were not cleared up until the publication, a few years ago, of Messrs Hurter & Driffield's photo-chemical investigations. A knowledge of their remarkable researches is so essential to a proper understanding of photography that an abstract of their papers is given here.*

111. Density, opacity, transparency.—They commence by asking, "What is a perfect negative?" Their answer is, that a negative is theoretically perfect when the amount of light transmitted through its various gradations is in inverse ratio to that which the corresponding parts of the original subject sent out. The negative is mathematically the true inverse of the original when the opacities of its gradations are proportional to the light reflected by those parts of the original which they represent.

In order that this definition may be understood, they explain the laws of absorption of light by black substances and define clearly the meaning which they attach to the terms opacity, transparency and density of a negative. The whole of their investigations depends upon these laws.

For substances, which do not reflect much light, such as black opaque bodies, or transparent coloured bodies, the relation between the light absorbed and the quantity of the substance present is very simple. If, between the eye and a source of light, we place a thin

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*Photo-Chemical Investigations and a new method of the determination of the sensitiveness of photographic plates, by Ferdinand Hurter, Ph.D., and V.C. Driffield. Journal of the Society of Chemical Industry, 31st May, 1890.

The Action of light on the Sensitive Film. Photography, 19th and 29th, Feb., 1891.

Relations between photographic negatives and their positives. Journal of the Society of Chemical Industry, 28th Feb., 1891.

Latitude in exposure and speed of plates. British Journal of Photography, 21st July, 1893.

The principles involved in enlarging, by V. C. Driffield. British Journal of Photography, 9th and 16th Nov., 1894.

layer of dilute Indian ink, that layer absorbs light and thereby reduces the intensity of the light transmitted. Assume that such a layer absorbs one-half of the light, then one-half of the light will be transmitted. Whatever may be the intensity of the original light, the intensity after passing this layer of ink will be one-half of what it was. The inter-position of two such layers will reduce the light to one-quarter of the original intensity, three such layers will reduce it to one-eighth and so on, each layer reducing the intensity to one-half of what it receives.

Had the first layer allowed $\frac{1}{3}$ of the light to pass through, then two such layers would reduce the intensity to $\frac{1}{9}$, three layers to $\frac{1}{27}$, etc. In general, any number of layers would reduce the intensity of the light to a fraction, which is equal to the fraction the first layer allows to pass, but raised to a power the index of which is the number of layers employed. If n equals layers employed and the first one reduced the intensity of the light to a fraction $\frac{1}{m}$, the n layers would reduce it to

$$\left(\frac{1}{m}\right)^n$$

If, instead of using so many layers, the first layer were made to contain as much Indian ink as the n successive layers contain altogether, we should find that the one layer now reduces the intensity of light by exactly the same amount as the n layers did. The reduction of the intensity is of course due to the black particles, and depends simply upon the number of them which are interposed per unit area. We can thus replace the number of layers by the number of particles and the law takes this form :—The intensity I_x of light after passing A molecules of a substance is a fraction of the original intensity I, such that :

$$\frac{I_x}{I} = \left(\frac{1}{C}\right)^A$$

For mathematical reasons, they express $\frac{1}{C}$ as a negative power of the base ε of the hyperbolic logarithms by making :

$$\frac{1}{C} = \varepsilon^{-k}$$

and they write :

$$\frac{1_x}{1} = \varepsilon^{-kA}$$

where k is the coefficient of absorption. The fraction $\frac{1_x}{1}$ represents and

measures the *transparency* of the substance. The inverse of that fraction, or $\frac{1}{1_x} = \varepsilon^{kA}$ measures the *opacity* of the substance. It indicates what intensity of light must fall on one side of the substance in order that unit intensity may be transmitted.

T being the transparency and O the opacity :

$$O \times T = 1$$

Density is quite distinct from opacity. By density, they mean the number of particles of a substance spread over unit area multiplied by the coefficient of absorption ; kA is what they term density and mark by the letter D.

In its application to negatives, the density is directly proportional to the amount of silver deposited per unit area, and may be used as a measure of that amount.

The relations between the three terms, transparency, opacity, and density, are the following :—

$$T = \varepsilon^{-D}$$
$$O = \varepsilon^{D}$$
$$D = log\ \varepsilon O = -log\ \varepsilon T$$

The density is the logarithm of the opacity or the negative logarithm of the transparency.

These relations hold good for some substances with regard to ordinary white light, for others only with regard to monochromatic light, and for others they do not hold good at all. Messrs Hurter and Driffield have satisfied themselves that they do hold good for the silver deposited as a black substance in negatives, so long as the silver does not assume a metallic lustre and reflects but a very small amount of light.

By means of these definitions we are now in a position to trace the connection between the densities of a theoretically perfect negative and the light intensities which produced them.

Since the density is the logarithm of the opacity, and since in a theoretically perfect negative the opacities are directly proportional to the intensities of the light which produced them, it follows that each density must be proportional to the logarithm of the light intensity which produced it.

The result is this : in a theoretically perfect negative, the amounts of silver deposited in the various parts are proportional to the logarithms of the intensities of light proceeding from the corresponding parts of the object.

The question arises, can such a negative be produced in practice?

In order to answer this question, it was necessary first to find a simple method of measuring the density of the silver deposited in negatives and then to study the influence of the developers upon the density of the deposits. The action of the light itself could then be investigated.

112. PHOTOMETER.—The instrument for measuring the density of the deposit is based on the relation existing between density and opacity. The opacity of the plate is measured, and in order to avoid calculations and references to tables of logarithms, the scale of the instrument is so arranged as to read the logarithm of the opacity, which is the density. The reason it is preferable to have the results expressed as density is because the density is a measure of the amount of silver deposited or of the chemical work done by the light.

The instrument devised by Messrs Hurter and Driffield is represented in Figs. 201, 202 and 203. It consists essentially of a small

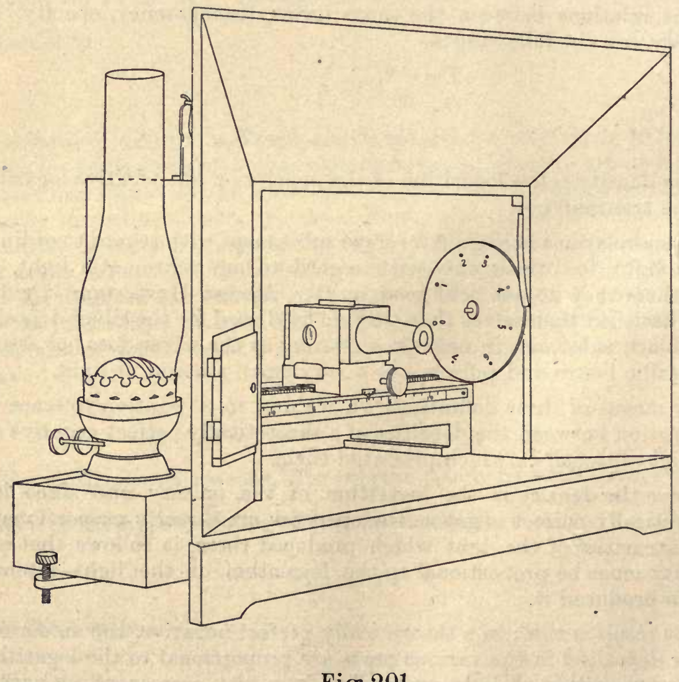


Fig. 201

Bunsen photometer, similar to those used for testing the illuminating power of gas, etc. The screen shown in section in Fig. 204 and

marked by a heavy black line, is a piece of paper with a grease spot in the centre about one millimetre in diameter; it is placed in a small cubical box or chamber. The chamber carries an eye piece, through which an image of each side of the disc can be viewed in two small

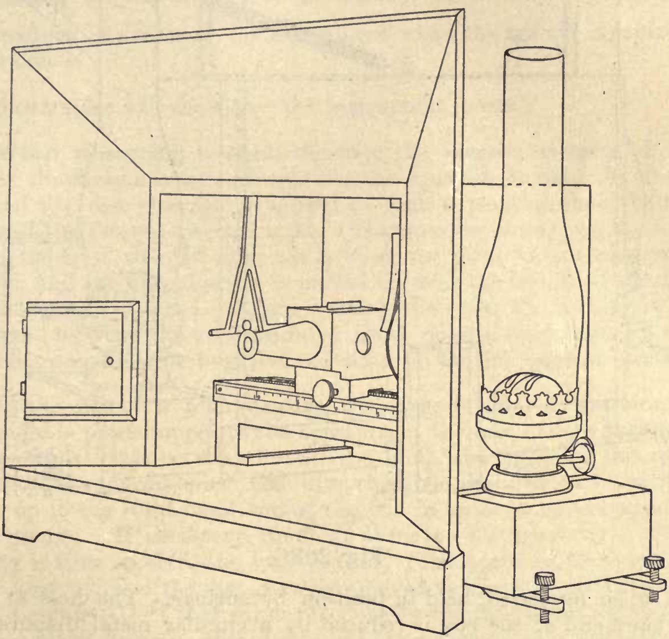


Fig. 202

mirrors, and so compared. The chamber can be made to slide in a straight line on a support by a rack and pinion. This arrangement is placed within a larger box, the ends of which have apertures through which light is admitted from two powerful petroleum lamps. Corresponding exactly with these apertures similar apertures are bored into the sides of the small chamber, which admit the light to either side of the Bunsen disc. The dimensions adopted for the larger box are 12 inches long, 6 in. high and 4 in. deep. The small chamber is a cube measuring 2 inches inside. Except the scales, everything inside of the box is blackened, and it is important to exclude all extraneous light by means of a screen. The heat of the lamps very soon injures the woodwork unless it is covered with asbestos cardboard and sheet metal.

The aperture in the left hand end of the large box is reduced to about $\frac{1}{4}$ inch diameter by a diaphragm. At this end is placed the

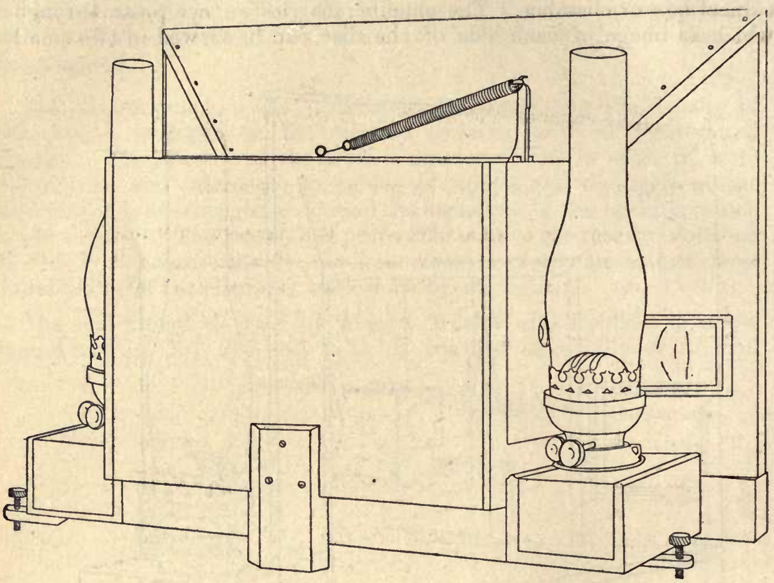
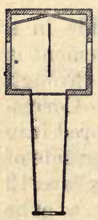


Fig. 203

plate to be measured, held in position by springs. The hole at the right hand end of the box is reduced by a circular metal diaphragm, revolving around its centre, and bearing seven circular apertures of various sizes.



The instrument is provided with a fixed scale which indicates the position of the disc chamber. It is constructed as follows: Let $2l$ be the length of the box between the two diaphragms, and x the distance of the disc from the middle point between the diaphragms. The numbers to be inscribed on the scale are given by the formula:

$$D = \text{Log.}\left(\frac{l+x}{l-x}\right)^2$$

Fig. 204

The centre of the instrument is marked with zero; the scale on both sides is symmetrical.

The circular holes in the revolving diaphragm are used for the purpose of reducing the light admitted through the aperture in the right hand

end of the box. B being the diameter of the largest hole, the substitution of another hole of diameter b reduces the area of the aperture, and, with it, the intensity of the light, in the proportion $\frac{B^2}{b^2}$, and the vulgar logarithm of the fraction $\frac{B^2}{b^2}$ is the density which is to be added to that read on the scale of the instrument when the hole B is replaced by the hole b.

Two examples will show how the instrument is used.

1. When measuring a small density, the largest opening of the circular diaphragm is turned opposite the aperture in the side of the box, and the disc chamber is moved to such a position that the two images of the Bunsen disc are alike. The number shown on the scale by the index of the disc chamber is read, the plate to be measured is inserted, and the disc chamber is moved towards the left until equality of the images is restored. Again reading the scale, the density is the difference between the two readings, those on the right hand of the zero being considered as negative and those on the left hand as positive.

2. In the case of a high density, the largest hole of the circular diaphragm is placed opposite the aperture of the box, and by inserting a piece of opal glass between it and the lamp, the light on the right hand side is reduced until the disc chamber requires to be moved almost up to the right hand end of the box in order to secure equality of the images. If necessary, the lamp is moved farther away. When equality is thus secured, the scale is read. The plate to be measured is then inserted and the disc chamber moved to the left until equality is again restored. If that cannot be done by the movement of the disc chamber alone, it can be obtained by using a smaller hole of the circular diaphragm.

Suppose the index stood at 1·10 on the right and afterwards at 1·55 on the left of zero, then the density would be $1{\cdot}10 + 1{\cdot}55 = 2{\cdot}65$.

If the index stood at 1·10 to the right and afterwards at 1·7 to the left, and equality could then only be restored by changing the largest hole of the circular diaphragm to one corresponding to a density of 0·7, then the density would be $1{\cdot}10 + 1{\cdot}7 + 0{\cdot}7 = 3{\cdot}50$. Higher numbers than 3·55 do not occur in ordinary negatives. A plate, the density of which is 3·55, only transmits $\frac{1}{3548}$ of the light which it receives.

The general rule for finding the density is: consider the numbers to the right of zero as negative numbers, those to the left as positive. Subtract the first reading from the second; the result is the density.

If the circular diaphragm be used as well, the amount it indicates must be added.

It will hardly be necessary to say that a plate of density 1, permits $\frac{1}{10}$ of the light to pass and that a plate of density 2, permits $\frac{1}{100}$ of the light to pass since 1 is the logarithm of 10 and 2 that of 100.

With this instrument, fairly accurate results are obtained. Messrs Hurter and Driffield tried it on mixtures of Indian ink and water, indigo solution and water, and many other substances. The greatest error did not reach four per cent, an accuracy quite sufficient for photographic purposes, where, from other causes still greater errors are liable to arise, as will presently be shown.

The lamps should be powerful petroleum ones with duplex burners. The flames should be on the planes at right angles to the axis of the instrument. Very erroneous results are obtained if Argand burners are used. The lamps should be placed close to the diaphragms, and it is advisable to provide a small stage outside of the diaphragm to hold coloured glasses, when a substance requires investigation in the light of a particular colour.

The importance of this instrument justifies the lengthy description given. It is for photographic experiments as indispensable, as the balance is in analysis.

113. DEVELOPMENT.—There is a generally accepted belief among photographers, that a great amount of control can be exercised in development over the density and the general gradations of a negative. With the plates experimented upon by Messrs Hurter and Driffield, they found that no such control existed.*

The plan adopted in carrying out these experiments was to subject pieces of one and the same plate to the varying conditions, the influence of which, on the density or the gradation, was the subject of investigation. A precaution always taken is never to develop a piece of a plate which has been exposed to the light without simultaneously submitting to the same developer a piece of the same plate which has not been exposed, and which is termed the fog strip. The object of this precaution is to ascertain exactly how much of the resulting density is due to the action of the light and how much is due to

* It is contended by some photographers that since the date of these investigations, such progress has been made in the manufacture of plates, that some, principally rapid ones, can, to a certain extent, be controlled in development. In their first paper, Messrs Hurter and Driffield pointed out the theoretical possibility of a plate being slow with one developer and rapid with another. They have recently given an instance of a plate which was three times faster with rodinal than with oxalate of iron.—E. D.

incidental fog, including therein fog inherent in the plate or caused by injudicious development, and also the density due to glass and gelatine.

In the series of experiments made to ascertain the influence of time of development and composition of the developer on the density, one-half of the plate was covered and the other half exposed to a standard light as will be presently more fully explained. After exposure, the plate was cut up in such a way that each piece included a portion of the unexposed and a portion of the exposed plate. Each strip was then developed, such modification in time of development or composition of developer being made, as formed the subject of investigation. The resulting intensities were then measured after fixing, washing and drying.

The first series of experiments was made with a normal developer containing pyrogallol ammonia and bromide of ammonia for the purpose of investigating the influence of the time of development. The times were 2·5, 5, 7·5, 10, 12·5 and 15 minutes.

The outcome of the experiments (Fig. 205) was that the total density grows with the time of development, but that the density due to light

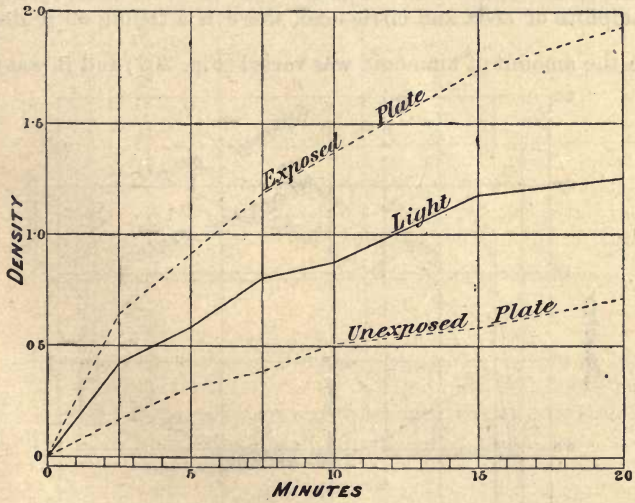


Fig. 205

reaches a limit in about 15 minutes. The continued growth of the total density is due to the action of the developer upon the bromide of silver, which had not been affected by the light.

In the next series of experiments (Fig. 206) the amount of pyrogallol was varied; they were repeated with the addition of sodium sulphite

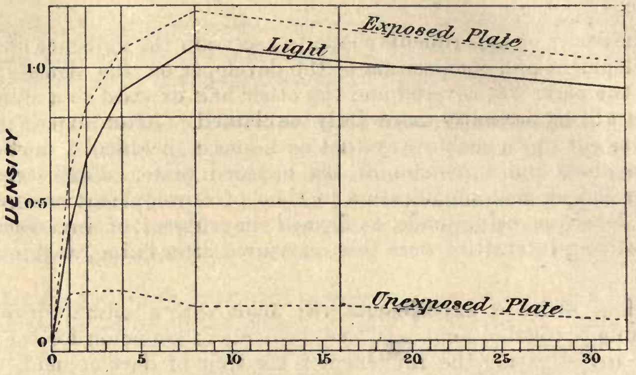


Fig. 206

and citric acid. They show that an excess of pyrogallol beyond a certain limit tends to retard development and the production of density. With sulphite of soda and citric acid, there is a falling off in density.

Then the amount of ammonia was varied (Fig. 207) and it was found

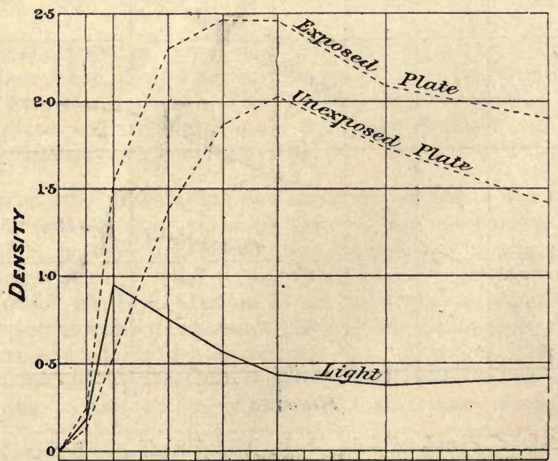


Fig. 207

that its addition, up to a certain extent, increases the density in a given time, but that the amount of ammonia which can be added

without giving rise to fog, and without simultaneously adding bromide, is very limited. The so-called accelerating action of ammonia being due almost entirely to its solvent action on bromide of silver, which, if the ammonia is increased sufficiently, results in greatly diminishing the density.

If, to any solution of silver bromide in very dilute ammonia, such as is used for development, bromide of ammonium be added, an immediate precipitate of bromide of silver is the result. The so-called accelerating action of ammonia, and the retarding action of ammonium bromide, are probably due entirely to this solvent action of the one, and the anti-solvent action of the other of these two reagents. The rapid production of fog, when ammonia is increased, is due to the fact that when pyrogallol solution is added to an ammoniacal solution of bromide of silver, the silver in solution is precipitated immediately in the metallic state.

The influence of the variation of ammonium bromide is shown in Fig. 208. Development in four minutes was entirely prevented when the amount of bromide was about ten times that of ammonia present.

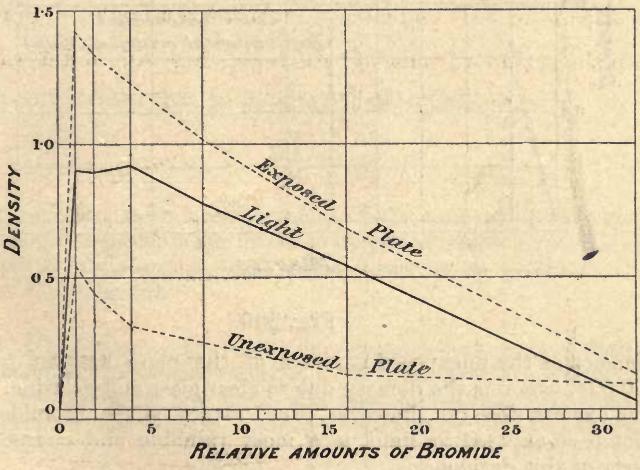


Fig. 208

It is interesting to point to Figs. 205 and 207, just to show the great amount of action which the alkaline developer may have on the bromide of silver, although it has never been exposed. This disagree-

able property is common to all alkaline developers, and it renders them unsuitable for scientific investigations. In all important work, the ferrous oxalate developer should be used for the reason, that it attacks unexposed bromide of silver so slowly that in one hour, and even more, no appreciable density can be developed upon a really good plate. Nor does its action vary much with its composition. The addition or omission of bromide from the constitution of this developer does not seem to have any great influence, and a greater or less concentration of the reagents within considerable limits, does not affect its action; indeed, no variation was found to arise from alterations in its composition, excepting the length of time needed for completion of development. Several experiments were made with various times of development and different compositions of the developer. The mean is shown in Fig. 209. In not one instance did

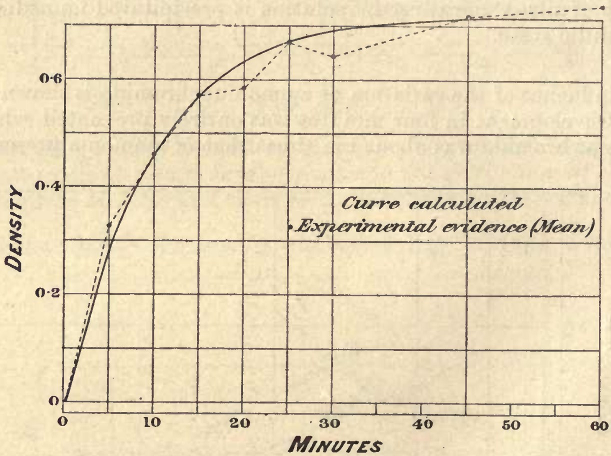


Fig. 209

the density of the unexposed portions of the plate amount to more than 0·098, which is the density due to clear glass and gelatine. That ferrous oxalate does not, however used, attack silver bromide which has not been exposed to light, is a most valuable and characteristic property of this developer.

An important result of this series of experiments is, that the density reached is dependent upon the time of development, as well as upon the exposure of the plate. The time it takes to reach a given density varies much with the gelatine employed in making the emulsion and the age of the plate, but with each plate it obeys a certain law, which

is more or less clearly visible in every experiment. It may be surmised that the number of particles of bromide of silver affected by the light is greatest in the front layer of the film, and decreases in geometrical progression as each succeeding layer of the film is reached, an idea which will be better appreciated when the action of light upon the film is explained. This idea expressed algebraically leads to the formula :—

$$D_t = D\,(1 - a^t)$$

where D_t is the density after t minutes development, D the limit of density reached by very prolonged development and t the time of development; a is a fraction depending upon the nature of the film, concentration of developer, temperature, etc. The relation of the calculated figures to the experimental data is seen in Fig. 209.

A very important conclusion can be shown to proceed from the formula representing the course of development.

If, on any one plate, two exposures were given, one of which would ultimately yield density D_1 and the other D_2, and if this plate were developed for a time t, then two densities, d_1 and d_2, would result so that :—

$$d_1 = D_1\,(1 - a^t)$$
$$d_2 = D_2\,(1 - a^t)$$

and it will be seen that on dividing these equations :—

$$\frac{d_1}{d_2} = \frac{D_1}{D_2}.$$

The resulting ratio is independent of the time of development, and is equal to the ratio of the ultimate densities which would be reached, so that the gradation of negatives appears to be independent of the time of development.

114. Gradation.—The above experiments have shown that with a well balanced developer, there is a limit to density, which depends upon the action of the light, and that, so far, the only control the photographer has lies in deciding whether he will reach that limit or not.

It has also become evident that if two different densities be developed upon the same plate to their extreme limits, the ratio existing between these limits must depend solely upon the action of the light. The question now to be considered is, whether it is possible by any modification of development to influence this ratio and whether this same ratio exists at all stages of development.

In making these experiments, the source of light adopted was a standard candle placed at one meter distance from the plate. A number of gradations were then produced upon the plate by exposing different portions of it to the light for different periods of time, always leaving one portion of the plate unexposed. It must be admitted that the candle is by no means an ideal standard, but there did not seem to be at the time of these experiments any satisfactory substitute, and if the suggestions for its use which are about to be made be adopted, its accuracy is quite sufficient for photographic purposes.

Assuming that the candle has been used before, it is lighted and the hardened tip of the wick snipped off with scissors; the flame of the candle will now be found to grow steadily in height, and as soon as the distance from the tip of the flame to the lowest point at which the wick blackens has reached forty-five millimetres, the exposure may commence. The candle flame may now be relied upon to remain sufficiently constant for about ten minutes, and that is amply long enough for our purpose. If, after this time, the light is required for any other purpose, it will be well to again trim the wick and start *de novo*. The height of the flame may be measured by a strip of card-board upon which two marks are made at a distance of forty-five millimetres. It is, of course, obvious that these experiments should be made in a room free from draught, and it is often a wise precaution to place the candle in a tall box, open on one side and well blackened inside. The candle should be well in view during the entire exposure, so that the operator may be aware of any fluctuation in the light. If the candle be used in the open room, all white or bright surfaces capable of reflecting light should be removed. The candle should be extinguished by an extinguisher and kept covered while not in use.

If a plate be examined by placing it between the eye and the red lamp, it will be found that the opacity of the film falls off at the edges. The edges should, therefore, be scrupulously avoided and strips for experimentation should be cut from the centre of the plate, or, at any rate, well away from the margin. The operation of cutting the plate should be conducted as quickly as possible, and as far away as possible from the red light, so as to avoid all fogging action of the light upon the plate. The width of the strips may conveniently be made about one inch.

In order to secure a constant ratio of illumination between the different exposures, and to be independent of any fluctuations that may take place in the light, the exposures are made behind a rapidly revolving disc *D*, Fig. 210, in which are cut sectors proportional to the exposures to be given. After inserting the plate in the dark slide, the latter is placed in its position behind the disc. The distance from the candle to the plate is carefully adjusted, and the candle is lighted and

trimmed. When the flame has reached the requisite height, the exposure may commence. The disc is caused to revolve, and, at a given moment, the slide protecting the plate is drawn, and the exposure con-

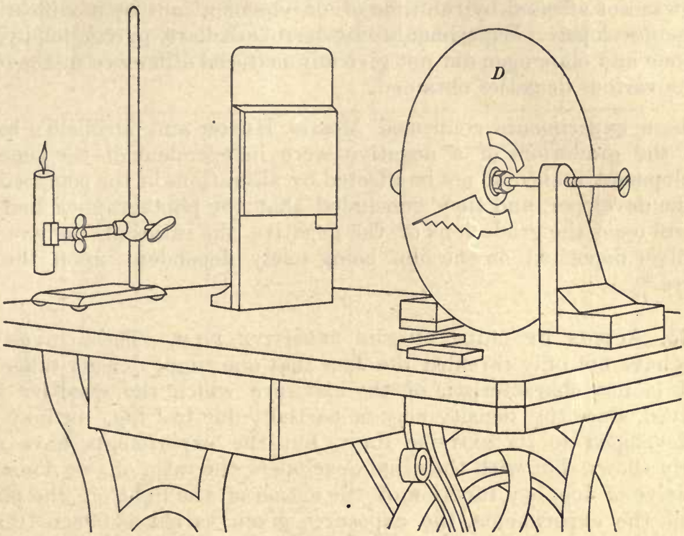


Fig. 210

tinued for the requisite length of time. The candle may be brought nearer or placed farther away from the plate so as to curtail or increase the exposure; at a distance of 0·707 metre, the light of the candle is equal to 2 candle meters, and at a distance of two metres it is equal to $\frac{1}{4}$ candle metre.

The unit of exposure adopted is the candle metre second, which is the exposure to the light of a standard candle at one metre distance during one second. The time may be measured with a watch or metronome. Within such limits as the experiments embrace, it has been ascertained that it is immaterial whether an exposure be made with a light of $\frac{1}{4}$ candlemetre for 40 seconds, or a light of one candlemetre for 10 seconds. It has also been proved by experiment, that as far as the ratios of densities are concerned, they remain constant whether the exposure be made with a candle, with a petroleum lamp or with daylight, so long as the product of intensity of light and time of exposure be the same.

Several experiments were made to show that the length of time of development does not affect the ratio of densities among themselves, but increases every density by proportional amounts. The results clearly showed that the ratio of densities was given by the light alone, and was not affected by the time of development nor by modifications in the developer. Experiments with ferrous oxalate, pyrogallol, hydroquinone and eikonogen did not give any material difference in the ratio of the various densities obtained.

These experiments confirmed Messrs. Hurter and Driffield's belief that the gradations of a negative were independent of the time of development, and could not be affected by alterations in the composition of the developer, and they concluded that the photographer had no control over the gradations of the negative, the ratios of the amount of silver deposited on the film being solely dependent upon the exposure.*

115. Action of light on the sensitive film.—These investigations have not only revealed the fact that one single density taken by itself is not characteristic of the exposure which the sensitive film received, since the density may be partially due to "fog," or may not be developed to its extreme limit, but the experiments have also clearly shown that with the usual developers, the ratio of two densities exclusive of fog, is a function of the action of the light on the plate. In all the experiments, the exposures given varied between 10 and 80 *c.m.s.* In tabulating the ratios found between the two exposures, it was discovered that the ratio, though constant for one particular plate, is very different for different plates. The ratio is, for the same exposures, smaller for rapid than for slow plates, but even with the same plate, the ratio between two densities varies for exposures which bear the same ratio to each other, but are different in absolute value. It is certain, therefore, that the ratio between two densities depends not only on the ratio of the exposures, but also on the sensitiveness of the plate and the absolute value of the exposures.

To discover the connection between exposure, sensitiveness and density produced, numerous experiments were made. The first investigation was as to the general effect of prolonged exposure on the density. A plate received exposures commencing with 0·625 *c.m.s.*, then doubling every time until 5120 *c.m.s.* The plate was developed with ferrous oxalate and measured. The results are represented in Fig. 211; the exposures being chosen as abscissae, the densities as ordinates. It will be seen that every time the exposure is doubled, the density increases, at first slowly, then considerably, and from 40 *c.m.s.* up to 1280 *c.m.s.*,

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*As already observed, it is contended by many photographers that this law no longer holds good with some of the plates of recent manufacture—E.D.

every time the exposure is doubled, nearly an equal addition to density is the result, the addition to density being on an average 0·266, but after an exposure of 1280 *c.m.s.* further doubling produces less and less increase in density. The first few densities are too small to admit of accurate measuring. From the figure it will be seen at once how rapidly densities grow at first as exposure is increased, and how slowly at last, densities tend towards a limit.

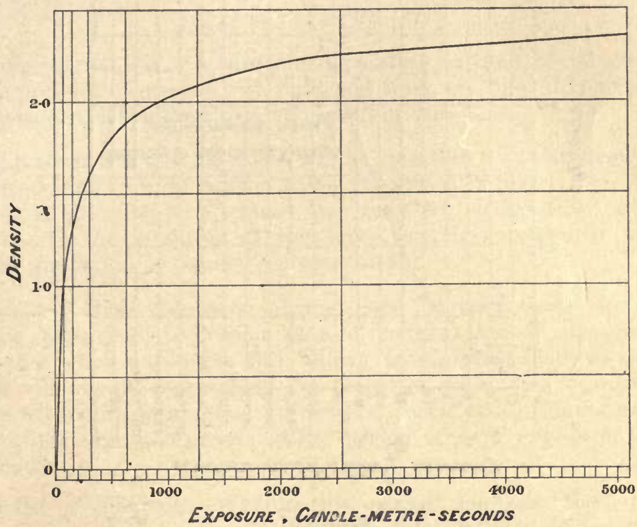

Fig. 211

If, in any part of the curve, the densities were proportional to the logarithms of the exposures, that portion of the curve should be discovered, if, instead of choosing the exposures as abscissae, the logarithms of the exposures were used. This is easily done when the exposures progress, as they do in these experiments, in a geometric series. Each new exposure has only to be marked equidistant from the previous one as abscissa. In this manner the results of another experiment with exposures from 1 to 524288 *c.m.s.* are plotted in Fig. 212. It will be perceived that the curve now consists of four distinct branches. It proceeds from exposure 1 in almost a horizontal direction, ascends slowly to exposure 16, from thence it proceeds almost in a straight line to exposure 2048, when the growth of densities becomes slow. The densities reach their maximum at exposure 16384 and from thence the curve returns, the densities diminishing slowly with increased exposures.

Four different periods are accordingly distinguished. The first period is termed the period of "under exposure;" it is comprised in the first curved portion. The second period, that during which the curve is

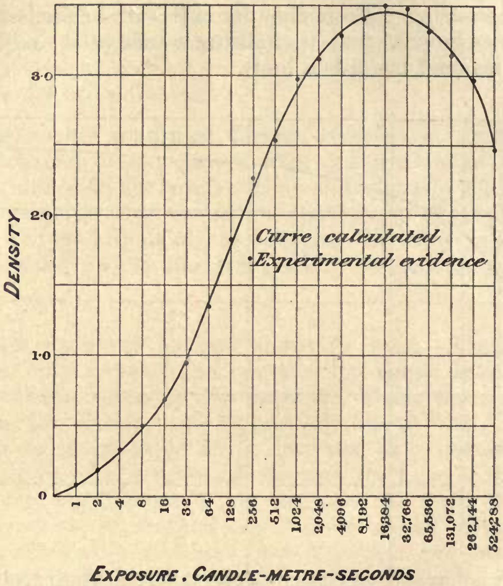

Fig. 212

almost a straight line, is the period of "correct representation." The third period is that during which the curve is again strongly bent as far as its maximum, this is the period of "over exposure"; and the last portion of the curve is the period of "reversal."

Period of Under Exposure.—During this period, the ratio between two densities is at first accurately equal to the ratio of the corresponding exposures. Of course, there is no defining point which marks the end of this period and the beginning of the next, but we learn from it that, for short exposures, the amount of silver reduced is directly proportional to the exposure.

Period of Correct Representation.—The second period of exposures has thus been named because, during this period, a plate is capable of giving a negative differing as little as possible from that which, at the beginning, was defined as theoretically perfect. That definition required that the densities of the negative should be pro-

portional to the logarithms of the exposures which produced them. It is characteristic of this period, that the densities are proportional to the logarithms of the exposures. This is shown in Fig. 212, where the densities are the ordinates, the logarithms of exposures are abscissae, and the period of correct representation a straight line. Dozens of plates were measured, and the densities falling within this period were found to conform to the very simple linear equation:

$$D = \gamma\,[\text{Log}\ It \pm C]$$

D being the density, γ a constant depending on time of development, It the product of intensity of light and time, *i.e.*, the "exposure," and C a constant depending upon the speed of the plate.

An answer can now be given to the question:—"Can negatives be produced such as were defined to be theoretically perfect?" And the answer is, they can be produced, but only by so carefully adjusting the time to the intensity of the light that the exposures may fall within the period of correct representation.

Period of Over Exposure.—Little need be said about this period. As the curve tends to become parallel to the axis of abscissae, it is clear that when exposures fall within this period, shadows and high lights will all be represented by densities which are almost equal. There will be no contrasts. In the first period, that of under exposure, the contrasts are too great; in the period of over exposure, they are too small.

Period of Reversal.—Within this period happens that peculiar phenomenon, the transformation of the negative into the positive, the "solarization," "reversal," etc. It is easy to understand how the negative becomes a positive. While the deep shadows still act upon the plate increasing the density, the high lights have passed their maximum, and their densities grow less and less. The more the exposure is prolonged, the less dense the high lights become, the shadows exceeding them in density.

The period of reversal, although very interesting, requires such enormous exposures that it need not be considered from a practical point of view. The three first periods, that of under exposure, that of correct representation and that of over exposure, are the only practically interesting portions of the curve.

From a clever and well-reasoned mathematical investigation, based on the idea that a certain definite amount of energy is needed to bring a particle of silver bromide into the condition in which it can afterwards be developed, and that it is only to the light absorbed by unaltered silver bromide that increase of density consequent on

increased exposure is due, Messrs. Hurter & Driffield deduce the following formula for the density of the plate after development:

$$D = \gamma \log_\varepsilon \left[O - (O-1)\beta^{\frac{It}{i}} \right]$$

in which O is the opacity of the unexposed plate, I the intensity of light; t the time of exposure, β a fraction the hyperbolic logarithm of which is $-\frac{1}{O}$, i a constant which is a measure of the slowness of silver bromide, and which they call the *inertia* of the plate, γ a coefficient depending upon the time of development, which they call the "*development factor*," and $\log_\varepsilon$ the hyperbolic logarithm of the expression between brackets. To illustrate the close accordance between their theory and experiments, the calculated curve is shown by a black line in Fig. 212, while the measured densities are indicated by dots. For this purpose, the opacity of the unexposed plate was measured for the rays of the spectrum from F to H and found to be 3.32. An inspection of Fig. 212 leaves little doubt that the action of light on the sensitive film is fairly represented by the formula, and consequently, it may be assumed as proved that the action of light at any moment is proportional to the amount of light absorbed by unaltered silver bromide.

To further elucidate this question, plates were prepared of different opacities, by spreading on equal areas different amounts of silver bromide. These plates were measured to ascertain their opacity to blue light, and the curves calculated which are represented in Fig. 213. The plates were then exposed and measured, the results being shown by dots.

It will at once be perceived that the more thinly the plates are coated, the shorter is that portion of the curve which is a straight line. This means that the period of correct representation is very short and great contrasts cannot be truly rendered by a thinly coated plate. It will also be found on closer inspection that the centre of the straight portion is in each curve in a different place, and that the thinner the plate, the shorter is the exposure necessary to reach the centre portion. This means that a thinly coated plate is somewhat faster than a thickly coated one, though they are made of the same emulsion. A thinly covered plate, however, appears very much faster than it is in reality. It is incapable of rendering wide contrasts, hence the negative always looks flat, and thereby gives to the eye the impression of over exposure. Thickly coated plates give also very much greater latitude in exposure. The plate illustrated by Fig. 212 would have given good pictures of subjects with contrasts varying from 1 to 20, though the

exposures had varied from 1 to 8, so that an exposure of 10 seconds, or one of 80 seconds, would have resulted in but little difference in the prints, but one of the negatives would have been much slower in

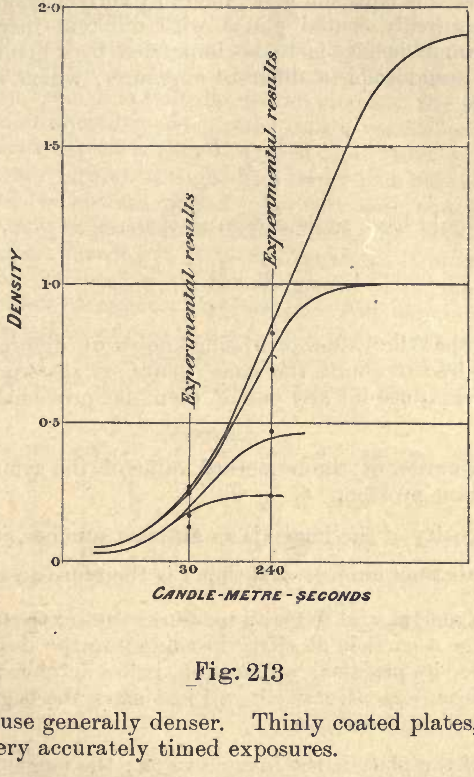

Fig. 213

printing because generally denser. Thinly coated plates, on the other hand, need very accurately timed exposures.

116. Speed of plates.—In the formula :

$$D=\gamma \log_{\varepsilon}\left[O-(O-1)\beta^{\frac{It}{i}}\right]$$

$O-1$ may be replaced by O when that represents a large number that is when the plate is richly coated and as $\log_{\varepsilon}\beta$ is $-\frac{1}{O}$, the equation can be transformed into another, viz. :

$$D=\gamma \log\left(\frac{It}{i}\right)$$

which equation holds good only when the numerical value of $\frac{It}{i}$ is greater than 1 and less than the opacity O. It is between these two limits only that this equation gives tolerably correct results.

Suppose two richly coated plates, with different inertias i and i_1, on which the same density is to be impressed by a given intensity of light I; they would require different exposures, which would have to be such, that:

$$\frac{It}{i} = \frac{It_1}{i_1}$$

or the times would have to be chosen so that:

$$\frac{t}{i} = \frac{t_1}{i_1}$$

This means that the values of i being known for different plates, the exposures required to obtain the same results are also known for those plates, if the exposure for any one of them has previously been ascertained.

The determination of the numerical value of the symbol i is therefore an important problem.

Since the density of the image is an abstract number, it follows that $\frac{It}{i}$ is also an abstract number, and that i is therefore an exposure. It was called the inertia, and it really measures that exposure which will suffice to change a particle of silver bromide into the developable condition. But for its practical application, it has another meaning. It measures the least exposure which will just mark the beginning of the period of correct representation.

The speed of the plate is the inverse value; the longer the exposure needed to bring the plate just to the beginning of the period of correct representation, the slower is the plate. Therefore, the speed of the plate is measured by the value $\frac{1}{i}$

The method adopted for measuring the value of i is as follows. The plate is given a series of exposures falling within the period of correct representation and then developed and measured, fog being deducted. The values of D and of the exposures It, permit of γ and i in the formula expressing the density being calculated.

It is preferable, however, to obtain the result by a graphic method, by means of which all calculations and references to tables of logarithms

are avoided. Printed diagrams are used similar to Fig. 212. The horizontal border is, like the scale of an ordinary slide rule, a logarithmic graduation for the exposures, but it is repeated four times instead of twice, as in the case of the slide rule. Vertical lines are drawn at the points 0·156, 0·312, 0·625, 1·25, 2·5, 5, etc., and they are divided into 25 equal parts, making the highest density 2·5 and the lowest 0. Having measured the densities and deducted from each the density of the "fog strip," which is that due to the incipient fog of the plate and to the glass and film, they are plotted on the vertical or exposure lines. A piece of black thread is then stretched along that part of the curve which practically forms a straight line, and which indicates the position and extent of the correct period. In this way, the position of the straight line may be ascertained before being actually drawn on the diagram. After drawing it, it is continued till it intersects the horizontal scale at the bottom of the diagram. The point at which the intersection takes place gives the inertia. The remaining points may now be connected by curves to the ends of the correct period line. The curve at the upper end represents the period of over exposure and that at the lower end the period of under exposure, the whole representing

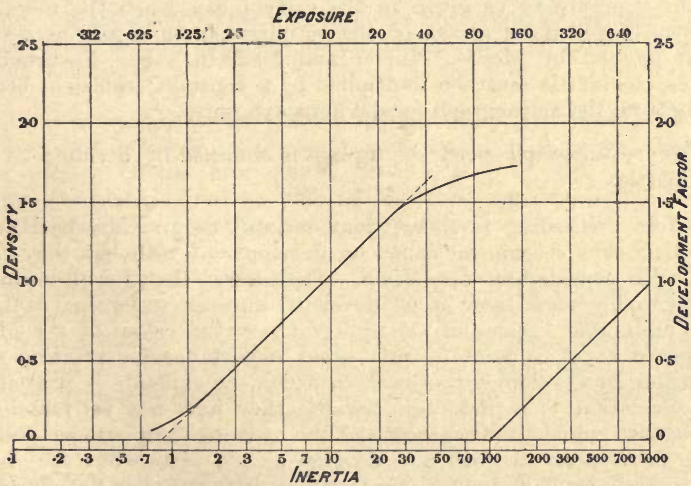

Fig. 214

the most characteristic features of the plate. The details just described will be better understood by a reference to Fig. 214, which shows the curve of an Edward's "isochromatic medium" plate.* The develop-

* This is not one of the diagrams of Messrs Hurter and Driffield's paper, but is introduced here as an illustration of the plate used on the Canadian Surveys.

ment factor is obtained by drawing through the point 100 of the exposure scale, a line parallel to the straight part of the characteristic curve, noting its point of intersection with the density scale. This gives the value of the tangent of the angle of inclination, which is the development factor. In the example given, the inertia is 0·85 and development factor 0·98.

When the inertia of the plate is known, it is possible to time the exposures in the camera so that the densities of the gradations are almost exactly proportional to the logarithms of the light intensities which produced them, and negatives can be produced which satisfy very nearly the definition given of a theoretically perfect negative. It must be borne in mind, however, that such a negative is not necessarily true to nature. If the negative is to be true to nature, a plate must be used which is richly coated, the exposure must be carefully timed, and the development must be carried only so far that the value of the development factor is numerically equal to one. On the other hand, such a negative would not itself generally give a print true to nature; but that subject will be treated later on.

The exposure to be given in the camera can, when the inertia is known, be found by means of the actinograph, an ingenious instrument devised by Messrs. Hurter and Driffield. For isochromatic plates, the inertia must be multiplied by a constant coefficient before using it on the actinograph for daylight exposures.

The "actinograph speed" of a plate is obtained by dividing 34 by the inertia.

Before proceeding further, it may be well to give Messrs. Hurter and Driffield's recommendations on development, although they have met with considerable opposition. They assert that, for all ordinary photographic work, there is no developer superior to ferrous oxalate. It is preferable, because of the uniformity of the colour of the silver deposited by it, a point of very great importance for printing and enlarging by developing processes, in which the exposure is arrived at by calculation; it is preferable, because they have not yet found one plate with which it disagreed, and this is more than can be said of other developers. It will also develop an old plate which may have been carelessly laid by for years; while, with another developer, it would be hopeless to obtain a passable result. It is preferable, because, of all developers, it is least liable to attack silver salts which have not been acted upon by the light, and because it will not lend itself to the production of foggy messes. It is not implied that other developers may not have their special uses; for instance, rodinal is of the greatest value in the case of certain plates when dealing with extremely short shutter exposures.

Proceeding with the operation of development, it is advisable that it be conducted at a fixed temperature, 65° Fahr. for instance. The developer itself should be brought to this temperature, and maintained at it by placing the developing dish in a water bath of the same temperature. The constituents of the developer are intimately mixed by stirring, and, at the moment of pouring on to the plate, the time is noted. The dish should only be rocked for a few moments, in order to expel any air bubbles from the surface of the plate, and should then be covered, so as to expose the plate no more to the red light than is absolutely necessary. Examination of the plate during development should be avoided as far as possible, as no red light whatever is safe in the case of even a fairly sensitive plate. It is important to know beforehand what development factor corresponds to a given time of development. This is found, once for all, by developing gradated exposures for different lengths of time and measuring the development factor in each case. For other times the factor is obtained by interpolation. Having decided on the density to be obtained and knowing the exposure, the density is plotted on the diagram on the proper exposure line, and the point obtained joined to the division of the horizontal scale representing the inertia of the plate. A parallel to this line through the point 100 of the bottom scale gives the development factor, from which the time of development is deduced.

After development, the plate is fixed in clean hyposulphite of soda and washed in the ordinary way. After washing, it is well to wipe the surface of the film gently with a plug of wet cotton wool. When the plate is dry, the back of it should be thoroughly cleaned and the film wiped with a silk handkerchief.

117. Negatives and positives.—Let a sensitive plate behind a negative be illuminated by a light of known intensity, and for simplicity's sake, let the negative have only two or three different opacities. Let the opacities of the negative be 40, 20 and 10; then we should expect the plate to be illuminated behind the negative with $\frac{1}{40}$, $\frac{1}{20}$ and $\frac{1}{10}$ of the original intensity of the light. Experiment reveals, however, that this is not so, but that the results of exposures to the light behind the negative are greater than those which would be produced by $\frac{1}{40}$, $\frac{1}{20}$ and $\frac{1}{10}$ of the original light intensity. The reason for this is not far to seek. When the light shines on the plate directly, say about 70 to 80 per cent of the light is reflected by the plate into space. When a negative is placed in front of the plate, the light is similarly reflected by the sensitive surface, but a considerable

portion of it is at once reflected back again by the two reflecting surfaces of the negative, so that behind a negative, less of the light transmitted by it is lost by reflection from the sensitive film, and consequently, more work is done on the film than would be the case if the same intensity of the light were to act upon a film free to reflect. The amount of light reflected by the sensitive film and back again by the negative depends upon the co-efficients of reflection of both the film and the negative; it may be foreseen that the same negative will give different results upon different printing surfaces, according to the amount of light which these surfaces reflect.

There is still another point to be considered. The opacity found with the photometer, is measured chiefly to the yellow rays of the lamp, whilst those rays are least active upon the plate. The opacity of the negative to the blue rays is, in all cases tested, greater than the opacity to the yellow rays.

These considerations explain why, when a negative is used for contact printing, its opacity must be considered as less than that indicated by the photometer; and when it is used for enlargement, the opacity must be considered as greater than that measured with the photometer, because, in the one case, the sensitive surface cannot reflect freely, whilst, in the other, it does so.

The exact amount by which the value of the opacity of the negative is to be increased or decreased depends, therefore, upon the reflection of the film, upon its sensitiveness to the different portions of the spectrum and upon the colour of the negative.

If it were not for these corrections, the relation between a negative and a positive could be at once deduced from the formula

$$D = \gamma \log \left(\frac{It}{i} \right)$$

In this formula, the intensity of the light, I, is reduced by the negative and must be divided by the opacity; which, in logarithmic calculation, means deducting the logarithm of the opacity from the logarithm of the intensity of the light. But, the logarithm of the opacity is the density measured by the photometer, which density is simply subtracted. In order to prevent confusion, the densities of negatives, including fog, are denoted by N, and the densities of positives by P. The equation which results, after introducing the correction a of the negative density for the reasons just explained, stands thus:—

$$P = \gamma \left[\log \frac{It}{i} - aN \right]$$

and this equation represents the general relation between a negative and its positive. P is the density of the positive produced behind the negative of density N upon a plate of inertia i by means of the light intensity I in the time t.

The coefficient a, which converts the density as measured into the printing density, is, for negatives developed by ferrous oxalate, usually a fraction; for pyro-developed negatives, it is generally nearly 1, if the negative be used for contact printing; but, when the negative is used for enlarging, this factor a, which changes the usual density of the negative into the enlarging density, is always greater than 1, even for negatives developed with ferrous oxalate.

The method for finding the printing factor a is illustrated by Fig. 215. Assuming that a speed determination has already been made of

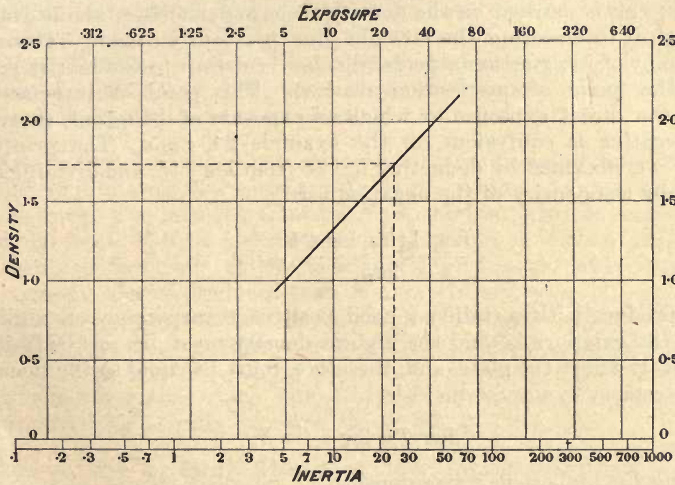

Fig. 215

the plate of which the printing factor is required, and that the extent and position of the line of correct representation are known, two more exposures are made, which, so long as they fall within the correct period, should lie as widely apart as possible. In the example, the exposures given are 5 and 40 *c.m.s.* Taking a negative of a uniform density of, say, about 1·0, the object now is to produce, through this negative, upon the plate under examination, a density which shall lie somewhere between the densities which will result from the two direct exposures already given. The geometrical mean of 5 and 40 is, roughly, 15; the intention is, therefore, to produce a density behind

the negative (the total density of which is, in the example, 0·985) equivalent to a direct exposure of 15 *c.m.s.* The necessary exposure is calculated thus :—

$$\text{Log } T = \log 15 + 0{\cdot}985.$$

$$T = 145.$$

This exposure of 145 *c.m.s.* is given through the negative and the plate developed together with that produced by the direct exposures of 5 and 40 *c.m.s.* Having measured the resulting densities and deducted fog, those densities which result from the two direct exposures, in the example 1·010 and 1·965, are marked upon the exposure lines 5 and 40 of a diagram. Through these two densities, a straight line is drawn which coincides with the correct period of the plate. In the example, the density produced through the negative is 1·730, exclusive of fog ; this is marked on the density scale, and a horizontal line drawn through it intersecting the straight line previously drawn. Through this point of intersection a perpendicular is drawn to the inertia scale and the point of intersection marked. This point of intersection gives the direct exposure to which an exposure of 145 *c.m.s.* through the negative is equivalent, in the example 24 *c.m.s.* The printing factor a is obtained by deducting log 24 from log 145, and dividing the result by the density of the negative used :

$$\frac{\text{Log } 145 - \log 24}{{\cdot}985} = {\cdot}79$$

To produce with certainty a good positive transparency on a given plate, the exposure behind the highest density must be just sufficient to slightly affect the plate, and, therefore, must be equal to the inertia. The necessary exposure would be

$$\log T = \log i + a\,N$$

N being the highest negative density.

But, the calculation need not be made at all. The printing density of the negative, measured on the density scale, is taken with a pair of compasses and the same distance to the right on the exposure scale measured from the inertia of the plate. The necessary exposure is read off at once.

It is undoubtedly a difficulty that with one and the same negative, its densities as measured photometrically have to be multiplied with different factors according to the plates used with it. For contact printing with negatives developed by ferrous oxalate, the factor varies from about 0·6 to 1·0. It is, however, sufficient for practical purposes

to use the factor 0·8, but, of course, it is always better to ascertain the correct factor by experiment.

It can be shown that one and the same negative is not equally suitable for all printing processes, and that a negative yielding a good ordinary silver print is generally incapable of giving a first-class enlargement on bromide paper. By variations in the time of development, it is possible to produce secondary negatives in which the scale of tones is either contracted or extended, and this function of development is of the utmost value in the production of special negatives for special printing processes.

118. Contact printing and enlarging.—The principles involved in contact printing and enlarging on paper are practically the same. In both cases, it is necessary that the negative shall have that range of gradations which the paper is capable of registering. The subject of contact printing has been treated separately in an able paper by Mr. V. C. Driffield*. It will be sufficient for our purposes to give his remarks on enlarging: with some slight and evident modifications, they may be applied to contact printing.

There is a prejudice against enlargements on bromide paper, many persons maintaining that satisfactory results cannot be produced on this material. It must be admitted, that the best possible results are probably obtained by a contact print in platinum or carbon from an enlarged negative; but at the same time, very much better bromide enlargements can be produced than one very often sees, if only, the conditions necessary for their production be observed. The failure, to produce satisfactory enlargements on bromide paper, is probably chiefly due to the fact that it is utterly impossible to produce a good enlargement from a negative which will yield a good contact print, and yet one negative is generally expected to serve both purposes.

There is also a general impression that the production of a good enlargement involves the use of a light of high intensity, preferably daylight, or, at any rate, such an illuminant as lime light. This impression is altogether a fallacy; on the contrary, given a suitable negative and a correct exposure, a perfectly satisfactory result can be obtained with artificial light of extremely feeble intensity. The mistake has arisen from attempting to produce enlargements from negatives of ordinary density with light of low intensity. The prolonged exposure necessary in such cases has not been realized, and under exposure has resulted; while, with daylight, it has been easy to secure a more adequate exposure and the better result has been

*The principles involved in the calculation of exposures for contact prints on bromide paper—British Journal of Photography of 22nd September, 1893.

attributed to the higher illuminant. But, neither daylight nor any other light will ever produce a satisfactory enlargement from an unsuitable negative. The only advantage, that appertains to the use of daylight, is that it does not necessitate the possession of an optical lantern; but otherwise, the balance of advantage is infinitely in favour of the lantern. Daylight is fickle, and more difficult to gauge accurately than an oil lamp; then again, it is only available in the day time, while enlarging by oil light can be conducted at any hour, day or night.

The essential conditions to consider in connection with enlargements are, firstly, the quality of the negative required; and, secondly, the estimation of correct exposure.

The consideration which determines the character of the negative suited for enlargement is the range of gradation which bromide paper is capable of yielding. The range of gradation of the negative must coincide with the range of gradation of the paper, if a satisfactory result is to be looked for. The range of the paper is the ratio existing between the two exposures, one of which just falls short of producing any deposit, and the other which just suffices to produce the deepest black which the paper is capable of recording when viewed by reflected light. Say, for example, that an exposure of five seconds just fails to produce a deposit and that one of 160 seconds just produces maximum blackness, the range of the paper would be as 5 is to 160, or as 1 is to 32. Now, obviously, for a negative to exactly correspond with the paper range, its capacity for transmitting light must have the same ratio—that is to say, the ratio between the opacities representing the deepest shadow and the highest light must also be as 1 is to 32. This means, that the difference between the maximum and minimum densities of the negative must be equal to the logarithm of 32, which is 1·505; but it must not be forgotten, that this is the difference of the printing densities which are obtained by multiplying the densities measured in the photometer by the printing factor a.

To investigate the range of bromide paper, a series of exposures was made and after developing, the gradations of the paper were measured by reflected light, employing for that purpose a modification of Hurter & Driffield's photometer. The readings do not, however, in this case express density, but the fraction of light reflected by the gradations as compared with light reflected by the normal white of the paper. A reading of 1·3 (log. 20) indicates that the gradation reflects $\frac{1}{20}$ of the light reflected by the pure white of the paper. Having measured the gradations of the entire strip in this way, they were plotted just as in

the case of a speed determination, Fig. 216 shows the result. It is at once seen that there is a strong resemblance between the paper scale and the characteristic curve of a plate; but, there is a peculiarity about the upper part of the curve. In the case of a plate, this would merge from the straight line of the correct period much more gradually: here, when the ordinate 160 is reached, there is no further measurable difference in the degree of blackness, so that, though exposures beyond

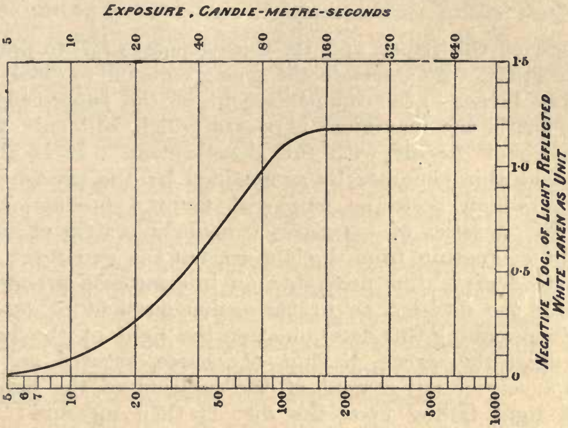

Fig. 216

160 *c.m.s.* produce slight differences just discernible to the eye, comparatively large density differences in the negative are represented by such trifling differences in the paper scale that they have simply to be ignored. The same argument applies to the very faintest indications of deposit at the other end of the scale, so that the practically useful range of the paper is less than a mere inspection of the gradations would lead one to infer. And, it must be remembered, that the test strip was practically developed out, while in picture making this is generally not the case. The figure, therefore, teaches that the practically useful range of bromide paper is capable of recording light intensities which are to each other as 5:160, or as 1:32; or, taking the difference between the logarithms of these numbers, in order to compare directly with the negative densities, it is at once learned that the density range of a negative suitable for enlargement must not exceed 1·5, and, as a matter of practical experience, a somewhat less range gives even better results. The figure further teaches that, as the more delicate intermediate tones of the negative correspond with the higher part of the paper curve, where the gradations approach each other

very closely; the small negative gradations should be kept as widely apart as possible. This means that a negative for enlarging should be fully exposed, so as to bring its lower densities into the period of correct exposure.

The reason for preferring a negative with a rather too contracted, than a too extended, range is that it is far better to make quite sure of getting the delicate half tones, even at some loss of the deepest black tones.

The range of the paper and its sensitiveness may, in practice, be found quite easily. To ascertain the sensitiveness is to discover what exposure just brings to the commencement of the paper range—that is to say, what is the maximum exposure which will only just produce evidence of deposit with full development. It is absolutely necessary that this exposure be ascertained by the precise source of illumination which is to be employed for the production of the enlargement. A series of exposures is made on a strip of the paper to the light emanating from the lantern, but the exposures progress by 1·41 instead of 2, thus providing an intermediate gradation, and so rendering the decision as to the commencement of range more exact. When making this determination the light of the lantern is projected upon the screen, having, of course, secured an accurate focus and a correct adjustment of the position of the light. The amount of light falling upon the disc is then measured and the exposures made.

A little difficulty may possibly be found in deciding which exposure must be regarded as marking the commencement of the range. It is always better, in case of doubt as to exactly where the range commences, to take too great rather than too small an exposure. It is advisable to be guided by the earliest unmistakable deposit, ignoring a possible one or two gradations of an extremely faint and indecisive character, and, while about to ascertain this information regarding the paper, it is well to mask a small portion so as to be able to judge of the normal whiteness of the paper. If, after development, there is evidence of fogging on the part to which light has not had access, the paper is to be discarded altogether; it will never produce a satisfactory enlargement. Such fogging may arise in the manufacture of the paper, or may be the result of deterioration from long or careless keeping.

It was ascertained that the first evidence of deposit on the paper to be used for enlarging was produced with an exposure to the light of the lantern of a certain number of seconds; it is transformed into *c.m.s.* by measuring the intensity of the lantern's light in candle metres by means of a simple shadow photometer. This, in the example given,

corresponds to 5 *c.m.* for the first evidence of deposit on the paper; it is one factor required before the exposure for the enlargement can be calculated. The other factor involves a consideration of the negative to be used, and is simply its maximum density multiplied by the enlarging factor a. From what has already been explained, it is clear that there is no one number which correctly expresses the density of a given deposit. It is perfectly obvious, that two deposits respectively developed with pyrogallol and ferrous oxalate may have identical visual densities, and yet, owing to the wide difference in their colour, their actinic densities will greatly differ; but further than this, it is now seen that a given deposit has three distinct density values,—its visual value, its contact printing value, and its enlarging value. The visual density of a deposit, developed with ferrous oxalate, which measures 1·0 becomes a contact printing density of 0·8 and an enlarging density of 1·4.

Let the negative to be used have visual maximum and minimum densities of 1·18 and 0·12 respectively; the visual density range is therefore $1{\cdot}18 - 0{\cdot}12 = 1{\cdot}06$. When, however, the densities come to be applied in the operation of enlarging, they require to be multiplied by 1·4. This converts the visual densities 1·18 and 0·12 into the enlarging densities 1·652 and 0·168, thus making the enlarging range of the negative 1·484. From what has been seen in considering the question of range generally, this negative, the enlarging range of which is so nearly 1·5, ought to produce a satisfactory enlargement.

The calculation of the exposure may now be proceeded with. All that is to be kept in view is that the exposure must be so timed, that the action of the light passing through the maximum density of the negative shall just bring to the commencement of the paper range: or, in other words, the exposure must be so timed that the light passing through the maximum density of the negative shall produce the same effect as an exposure, without the interposition of the negative, of 5 *c.m.s.* The maximum enlarging density of the negative is 1·652, and as this density only transmits about one forty-fifth of the light it receives, it is obvious that, in order to produce the same result upon the paper as would be produced by an exposure of 5 *c.m.s.* to the naked light, it is necessary to multiply 5 *c.m.s.* by forty-five, the opacity corresponding to the maximum density. Five times forty-five is 225, and this is the exposure required.

This calculation is more conveniently made as follows:

Maximum enlarging density of negative.............	1·652
Log 5, exposure marking commencement of range...	0·699
	2·351

The number corresponding to this logarithm, 224·4, say 225, is the exposure required in *c.m.s.*

The factor 1·4 for converting the visual into the enlarging density, applies only in the case of negatives developed with ferrous oxalate. In the case of negatives very yellow in colour as from development with unpreserved pyrogallol, this factor will be somewhat greater; how much, will be easily decided by an experiment or two.

CHAPTER VI.

Photographic Operations.

119. Dry plates.—For surveying purposes, photographs must be clear and full of detail; the points to be plotted must be well defined and easily recognized. Just at the start, a serious difficulty is met with; distant points, which are those wanted by the surveyor, are always more or less indistinct in a landscape, the distance merging into a uniform tint by the effect of what is known as "aerial perspective." This effect is due to the light diffused by the mass of air between the observer and the distant point; the greater the distance, and consequently the greater the mass of intervening air, the more light is diffused and the more indistinct the distance becomes. From the artist's point of view, aerial perspective is necessary to give the impression of distance; but to the surveyor, it is most objectionable.

The light diffused by the atmosphere is that which it has previously absorbed, and consists mainly of rays of shorter wave lengths, which, although not very luminous, have the strongest actinic power. It thus happens that aerial perspective is very much exaggerated when translated by photography, the strong effect of the light emitted by the blue haze, through which the distance is seen, completely blurring on the plate the details of the image, so much so, that the photograph becomes almost useless for surveying. Several causes contribute to the same result, such as the presence of smoke in the air, dust raised by wind, etc.

To get rid of this effect, it is necessary to have a plate which shall not be acted upon by the rays of greater refrangibility. Ordinary plates are, within the limits of exposure given in the camera, sensitive to the blue and violet rays only, but orthochromatic plates are made which are acted upon by the other end of the spectrum, although the maximum of sensitiveness is still in the blue violet region. By using a screen of a deep orange tint, it is possible to cut off nearly the whole of the rays further than the green: such a screen in connection with orthochromatic plates furnishes a partial solution of the difficulty. It is, indeed, not entirely removed, because air absorbs, and therefore diffuses, not only blue and violet rays, but also those of lesser refrangibility; the effect of aerial perspective still exists, but instead of being exaggerated by photography, it is reduced.

All that is required in the plate is, that it shall be sensitive to other rays than blue and violet, because these rays are cut off by the screen. It is not necessary that it should be acted upon by the red rays, in fact, it is preferable for convenience of manipulation, that it should not.

Fig. 217 represents the action of the spectrum on "Edward's isochromatic medium" plates, exposed behind a deep orange screen. The action of light commences at *D* and nearly ends at *b*. Between *b* and

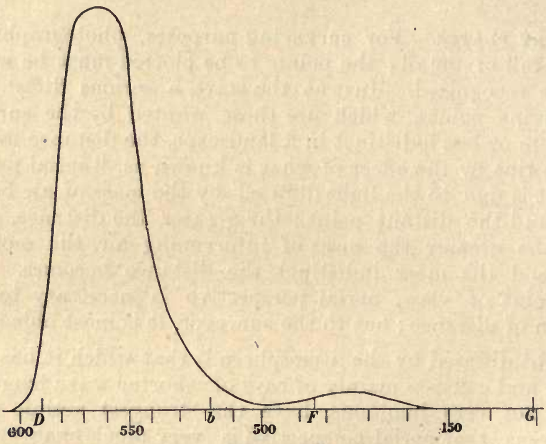

Fig. 217

F there is very little action; just beyond *F*, is a band of slight sensitiveness. It will be seen that the image on this plate is formed entirely by the yellow and green rays; consequently, the photograph may appear quite unnatural. Yellow photographs as white; an autumn landscape when the leaves have turned yellow, looks as if the trees were covered with snow.

The use of orthochromatic plates is not without its drawbacks. The shadows, in a landscape, receive their light from the sky and it is this light, reflected into the camera, which forms the image. But the light of the sky is mostly blue and violet and does not act on the plate behind the orange screen; the result is that the shadows are much more intense than on ordinary plates.

In order to find the proportion between the light coming from an object in direct sunlight and that coming from the same object when placed in the shade, Messrs. Hurter and Driffield photographed a folding screen, upon each of two folds of which had been placed a sheet

of white cardboard and a sheet of matt black paper. The screen was so placed that one fold was illuminated by direct sunlight, and the other by the light reflected from the sky. They cut a plate in two and exposed one half in the camera to the screen; on the other half they made a series of candle exposures. Both halves were developed for the same time, and the densities measured and plotted. They found that the densities of the black and white paper in the image of the screen corresponded to the following candle exposures on the characteristic curve plotted from the other half of the plate :

White in sun.	22·50
" shade.	10·20
Black in sun	1·62
" shade	0·77

The direct illumination from the sun was a little over twice that from the sky.

The same experiment repeated with an orthochromatic plate behind a deep orange screen gave the illumination as twelve times greater in direct sunlight than in the shade. It is at once seen, what an enormous difference there must be between a negative taken under these conditions and one taken on an ordinary plate, and how much more intense the shadows are in the first photograph.

The proportion between direct sunlight and skylight varies with the altitude of the sun and with the absorption of the atmosphere : the less light absorbed, the greater is the contrast. Shadows look more intense when the sun is high than when it is low, but it is in the mountains, at high elevations, that the contrast is greatest. In general, the air is very pure and the coefficient of absorption small; only rays of very short wave lengths are absorbed and diffused, and the sky assumes a strange deep blue tint. The thickness of the overlying atmosphere being so much less than at sea level, the absorption is still further reduced and the shadows of the landscape become very intense. The effect is exaggerated by the orthochromatic plates with deep yellow screen.

It is now easy to understand why good photographs of Alpine scenery are so scarce: the wide contrasts present, ranging from snow in sunlight to dark pines in shade, and the intensity of the shadows, combine to make them the most difficult subjects to photograph. Satisfactory results cannot be expected unless the very best plates are used.

Fortunately, Messrs. Hurter and Driffield have shown how to recognize a good plate. There must be silver enough on it to give a long period of correct representation; any plate, in which the straight part

of the characteristic curve is short, must be at once rejected. The plates now used here are double coated "Edward's isochromatic medium," specially made for the purpose, and backed with a non-actinic coating for preventing halation.

When a subject presenting strong contrasts is photographed and a long exposure given, it is observed that the action of the light appears to spread upon the plate. The edge of the high lights, instead of ending abruptly, merges into the shadow by a gradually decreasing tint more or less extended, according to the intensity of the high light and the length of exposure. This is called halation, and is caused by the light which has passed through the film, struck the posterior surface of the glass plate, and been reflected by it to the back of the film. The fog which is seen on over-exposed plates is due, in part, to halation. The remedy is to stop the light when it reaches the back surface of the plate by coating it with some black or non-actinic material that will absorb the light. Any kind of opaque material will not cure halation. Two conditions are requisite: firstly, the coating must be in optical contact with the glass; a black cloth, for instance, pressed on the plate, would not produce the least effect on halation; secondly, the refractive index of the coating must be the same as the index of glass; otherwise there will still be reflection at the surface of the glass.

The backing is wiped out with a wet sponge before developing.

Dry plates must be preserved from heat and damp. It is advisable to place the boxes, in which they are received from the manufacturers, into a larger tin box securely locked. They are inserted in the plate holders in weak ruby light, and are then dusted with a flat camel hair brush. Two or three strokes of the brush are sufficient; more would develop electricity on the film which would attract the dust. The plates are then numbered in pencil in one corner; it is usual here, to write on the plate the number of the dozen and the number of the holder. After being exposed, the plates are removed from the holders and packed face to face in the original boxes, without anything between the plates. While unpacking and packing the plates, it is convenient to have grooved metal boxes into which the whole contents of a box are placed, and from which they can be handled more easily, both in filling and emptying the holders.

Films were tried here before glass, but the results were not satisfactory. Admitting that films could be made as good as glass plates, and it does not seem that this point has yet been reached, their only advantages over glass would be freedom from breakage and lessened weight; there is no material difference in the bulk of the holders to be carried by the surveyor, whether he uses glass or films.

On these points, it may be said that small glass plates do not break; that is the experience on the surveys here and they are being made in as rough a country as will be found anywhere.

The surveyor carries with him, at the utmost, eighteen plates; if they are half plates ($4\frac{3}{4} \times 6\frac{1}{2}$ in.) their weight is not a serious consideration. The difference in weight between glass and films may become an important matter when the supply for the whole survey is considered, but the conditions must be very exceptional indeed when means cannot be found for the transportation of the weight represented by the plates.

120. Exposure.—The exposure to be given to a plate is inversely proportional to the intensity of the light by which the subject is illuminated. A subject requiring an exposure of ten seconds when the intensity of the light is one, will, all other conditions unchanged, require an exposure of five seconds with a light of intensity two. Before giving any rules for exposure, it is therefore necessary to investigate the variations of daylight.

The light received by an object in direct sunshine consists of three parts: firstly, the direct rays of the sun; secondly, the light diffused by the sky; and thirdly, the light reflected or diffused by surrounding objects. In a landscape and for our purposes, the third part need not be considered.

The light of the sun, after passing through the atmosphere, has lost a portion of its constituent rays, the loss being greatest for the radiations near the violet end of the spectrum and smallest for those near the red end. Taking as a unit, the intensity of a radiation in the light of the sun just outside of the atmosphere, and denoting by a the fraction of this radiation remaining after the light has passed through a thickness of one atmosphere, the intensity, after passing through a mass of air of M atmospheres, is:

$$I = a^{M}.$$

M may be taken as proportional to the secant of the sun's zenith distance up to 65°: for greater distances it becomes:

$$\frac{0 \cdot 0174 \times \text{tabular refraction}}{\text{cosine apparent altitude}}$$

The coefficient of transmission, a, is subject to great variations according to the greater or less transparency of the atmosphere: hence, it is quite impossible to predict accurately what the intensity of the

light will be when the sun is at a certain altitude, but when it is not too low, the intensity may be found with sufficient accuracy for photographic purposes.

Edward's orthochromatic plates behind a deep orange screen are, it has been shown, sensitive to the rays from D to b, the greatest action being about $0 \cdot ^{\mu}560$; for exposing these plates, it is only the intensity of these particular radiations which has to be considered and investigated. An examination of the results obtained by different observers shows that for $0 \cdot ^{\mu}560$, the coefficient of transmission through one atmosphere at sea level and under average conditions of the air, is about $0 \cdot 65$; at an elevation of 10,000 feet, with dry and pure air, it averages at least $0 \cdot 80$. These are the two coefficients upon which subsequent calculations are based.

According to Clausius' theory of the diffusion of light, the intensity of the light reflected and diffused by the sky is expressed by the formula:

$$S = Z \cos z \, (1 - a^{M})$$

in which S is the light from the sky, Z the ratio of the lost portion of the sun's light to that reflected and diffused by the sky, z the zenith distance of the sun, a and M the coefficient of transmission and air mass respectively.

The values given by this formula are too large and do not agree with the results obtained on orthochromatic plates. To elucidate this point, series of gradated exposures were made on white blotting paper, in direct sunshine and in the shade, at various altitudes of the sun. The characteristic curves were plotted and, for each altitude, two inertias were obtained, one for direct sunshine and one for skylight: the intensities were in the inverse ratio of the inertias. The numbers so obtained were used for calculating the total light received by a surface exposed normally to the sun's rays, the light coming directly from the sun being computed by the formula already given,

$$I = a^{M}$$

with the coefficients of transmission $0 \cdot 65$ for sea level and $0 \cdot 80$ for 10,000 feet altitude.

The results are represented by Fig. 218 in which the abscissae are the altitudes of the sun. The ordinates are, for convenience, made equal to the reciprocals of the intensity of light, that is to the fraction $\frac{1}{I}$ to which the exposures must be proportional. The unit adopted is

the intensity at sea level when the sun is at the zenith. There are two curves, one corresponding to sea level and the other one to an elevation of 10,000 feet; it will be noted, that there is very little change in the exposures required at great elevations until the sun approaches the horizon: this agrees with the experience of our surveyors. The curves of Fig. 218 agree fairly well with the results obtained on orthochromatic plates; there are occasionally large discrepancies, but no more than must be expected. The transparency of the air varies so much that no formula can express accurately its absorption.

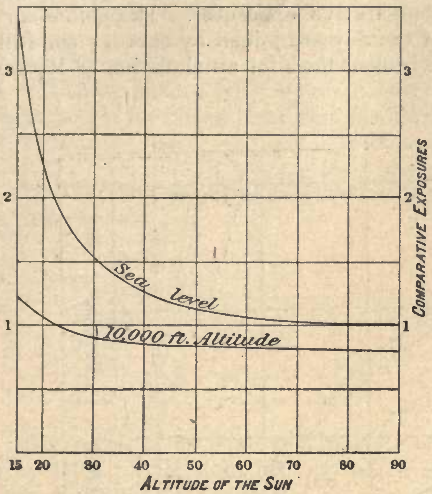

Fig. 218

A number of actinometers have been devised for the use of photographers. The most popular depend upon the action of light on specially prepared bromide paper, the intensity of the light being measured by the time of exposure required to produce a certain tint on the paper. It is hardly necessary to say, that these instruments cannot be depended upon for ascertaining the exact exposure to be given to orthochromatic plates behind a deep orange screen. The prepared bromide paper is acted upon by the rays beyond F only, while the orthochromatic plate answers only to the radiations between D and b. For the indications of the actinometer to be accurate, the proportion between the different radiations in daylight should not vary; but that is not the case—the proportion is changing constantly. If, the exposure found by the actinometer were correct with the sun high in the sky, it would be excessive with a low sun. The same remark applies to all actinometers: every one indicates the intensity of some particular radiations which are not those acting upon the orthochromatic plate behind a deep orange screen, nevertheless, an actinometer may occasionally prove useful.

The next step to be taken is to find the inverse of the intensity of light for every hour of the day, and for every day of the year, at the place where the photographs have to be taken. The altitudes of the sun are calculated for the latitude of the place and for one day in every month, and the corresponding values of the fraction $\frac{1}{I}$ are taken

from the curves at Fig. 218. The results for 50° of north latitude are plotted in Fig. 219; the abscissae are the months and days of the year, and the ordinates the comparative exposures. The points corresponding to the same hour of the day are joined by curves; the full lines are for sea level and the broken lines for an elevation of 10,000 feet.

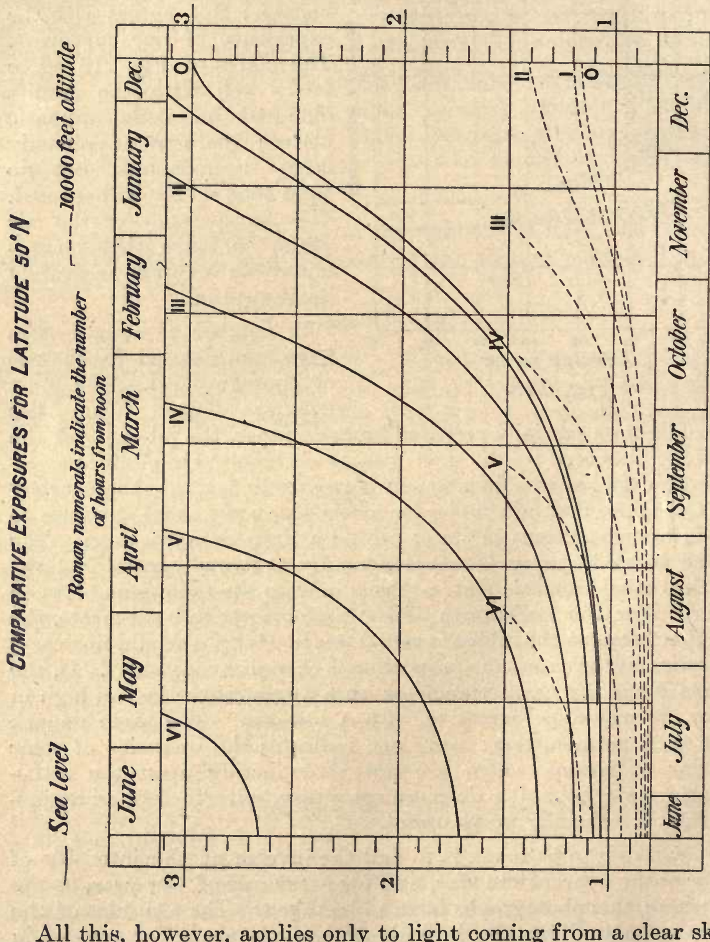

Fig. 219

All this, however, applies only to light coming from a clear sky; an allowance has to be made when the sun is obscured by clouds. Messrs. Hurter and Driffield, in the instructions for their actinograph, adopt five degrees of brightness: very bright, bright, mean, dull, and very

dull. Very bright light is that coming from a pure sky, and mean is when there is just sufficient sun to cast a very faint shadow. Very dull is the dullest light in which it would be at all reasonable to take a photograph, a definition which, it must be admitted, is somewhat indefinite, but which a little practice will make clear. Bright is between mean and very bright, dull is between mean and very dull. Taking as a unit the exposure with a very bright light, the exposure for bright light is $1\frac{1}{2}$, for mean light two, for dull light three, and for very dull light four.

With orthochromatic plates and orange screen, the proportions are as follows:—

Light.	Exposure.
Very bright	1
Bright	1·5
Mean	2
Dull	4
Very dull	8

The increase of exposure in dull weather is due to the bluish gray colour of the light coming from a cloudy sky; the blue rays being stopped by the orange screen, do not reach the plate.

We must now find the unit of exposure, or the exposure required at sea level, when the sun is at the zenith; with this unit, the diagram of Fig. 219 gives at once the exposure at any time. Under ordinary circumstances, there would be a different unit for each kind of subject; thus, for views of the sea the unit would be small, while for landscapes with dark foliage in the foreground, it would be large. In surveying, the character of views and the scale of tints to be reproduced are tolerably uniform, and it will generally be found practicable to adopt one unit for the whole of the work.

Let us consider Alpine scenery under a bright sun, snow being present in quantities; the scale of tints to be reproduced extends from snow in sunshine to dark trees in the shade of the valleys. These extreme contrasts being exaggerated by orthochromatic plates with orange screen, it is easy to understand that unless the plate be given all the exposure that it will stand, the shadows will be hopelessly underexposed. It seems, therefore, that the exposure should be proportional to the intensity of the high lights, reversing the old rule of exposing for the shadows. The excessive contrast between snow in sunshine and snow in the shade, may be decreased by making use of the period of over-exposure in such a way, that snow in the shade and the brightest rocks shall be rendered by densities corresponding about to the end of the period of correct representation. An indirect

advantage of this mode of procedure is to increase considerably the range of the plate and to secure better detail in the shadows.

Bearing these remarks in mind, the unit of exposure is ascertained by trial plates on a landscape of the same kind as those of the survey. The object is to find the greatest exposure which the plate will stand. Let us suppose that on the 20th of September, at 2 p.m., and at sea level, the proper exposure is found to be 34 seconds. The diagram of Fig. 219 gives for this day and hour 1·4 as the coefficient by which the unit of exposure must be multiplied : the unit is, therefore, 34 : 1·4 or 24 seconds. It is advisable to repeat the experiment on several days, so as to obtain a good average unit, because the intensity of daylight is subject to great fluctuations.

Suppose now, that it is required to find the exposure at 4 p.m. on the 20th of August. The diagram of Fig. 219 gives, for sea level, 1·65 times the unit, or 40 seconds. For an elevation of 10,000 feet, it would be 0·92 times the unit, or 22 seconds. For any intermediate elevation, the exposure can be obtained by interpolation.

The surveyor, however, does not make these calculations every time he has to expose a plate : before starting for his day's work, he examines his diagram and notes the exposures for that particular day and for the elevation of the ground. These exposures may, during the course of the day, be increased to suit the light, when the sky is not clear ; a view that is generally dark also requires a longer exposure.

If desired, a new diagram like Fig. 219 may be plotted with the unit of exposure found, giving directly the exposure instead of the factor by which the unit must be multiplied.

If, it were attempted to give such prolonged exposures and to handle the plate in accordance with ordinary practice, the negative would be flat and foggy and present all the well known characteristics of over-exposure. It is imperative to pay particular attention to the following points :—

1st. The plate must be backed, or it will be ruined by halation.

2nd. The inside of the camera must be dead black and provided with one or two diaphragms. The sides of the camera are illuminated very strongly by the light diffused by the plate, and also by some of the rays coming through the lens : if the sides were not dead black, this light would again be reflected on the plate which would be fogged.

3rd. With the same object of preventing reflections inside of the camera, the diameter of the back lens of the object glass must be small. This condition is realized in Zeiss' anastigmat F/18.

4th. The light, which does not contribute to form the image on the sensitive plate, must be prevented by a hood from entering the camera; otherwise, some of it would be reflected or diffused on the plate, and cause more or less fog.

5th. Under no circumstances must the sun shine on the lens; when the line of sight is somewhere in the direction of the sun, it may be advisable to cut off the sky, if very bright, by closing the upper part of the hood's opening.

So far, our remarks have been confined to Alpine scenery, but, all views taken for surveying purposes being distant landscapes, the rules given are applicable to all cases.

It is useful to know the portions of the characteristic curve which are brought into play by the exposures given. For this purpose, a plate is given a series of gradated exposures to a steady light, like that of a lamp which has been burning for some time. The plate is developed, measured, and its densities plotted (Fig. 220).

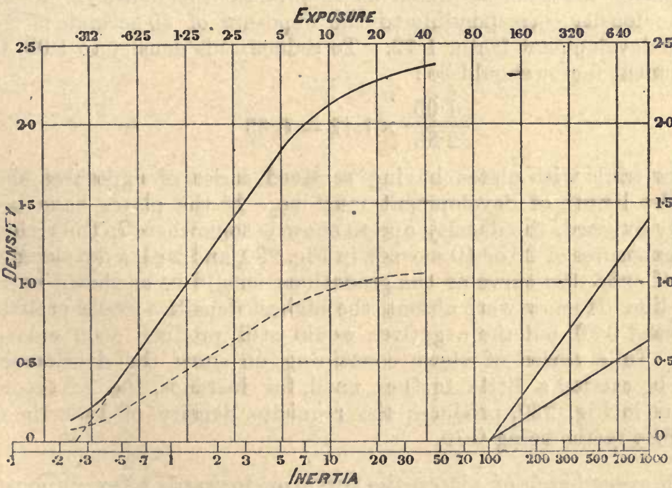

Fig. 220

In the same dish and at the same time, a landscape exposure is developed: its densities are measured and compared with the characteristic curve.

121. Development.—The backing having been washed off, the plates are put in grooved trays containing one dozen each, and are

developed with freshly prepared iron oxalate. The formula used here is:

Oxalate of potash	1 oz.
Water	3 ozs.
Bromide of potassium	15 grs.
Acetic acid	10 mins.

And:

Sulphate of iron	1 oz.
Water	2 ozs.
Acetic acid	2 mins.

To each ounce of the oxalate solution are added two drams of the iron solution.

The development is carried out for a definite length of time, so that the greatest densities of the negative be those adapted to the enlarging process. Assuming, for instance, that bromide paper is used, and that the highest negative density required is 1·05, the development must be so timed as to produce this density. In Fig. 220, the density corresponding to the exposure of 40 seconds is 2·35 and the development factor 1·42. To reduce this density to 1·05, the development factor should be:

$$\frac{1{\cdot}05}{2{\cdot}35} \times 1{\cdot}42 = 0{\cdot}63$$

A few trials with plates having received series of exposures show what the length of development must be. If the plates have been properly exposed, the density due to snow is somewhere in the vicinity of the exposures of 20 or 40 seconds in Fig. 220, and with a development factor of 0·63, the curve of the gradations should be as shown by the dotted line. If snow were absent, the highest densities would probably not exceed 0·90, but the negatives would still produce good enlargements. In a series of views containing no snow, the development should be carried a little further, until, for instance, the 7·5 seconds exposure in Fig. 220, produces the requisite density of 1·05, the development factor being 0·75.

With every batch of plates, the surveyor forwards a few duplicates which are developed separately, for checking both the exposure and the time of development.

After developing for the requisite time, the developer is emptied out and the tray filled with water several times; the plates are then taken out and put in another grooved tray containing hyposulphite of soda. When fixed, they are washed and before putting them to dry, the film is wiped with a tuft of cotton.

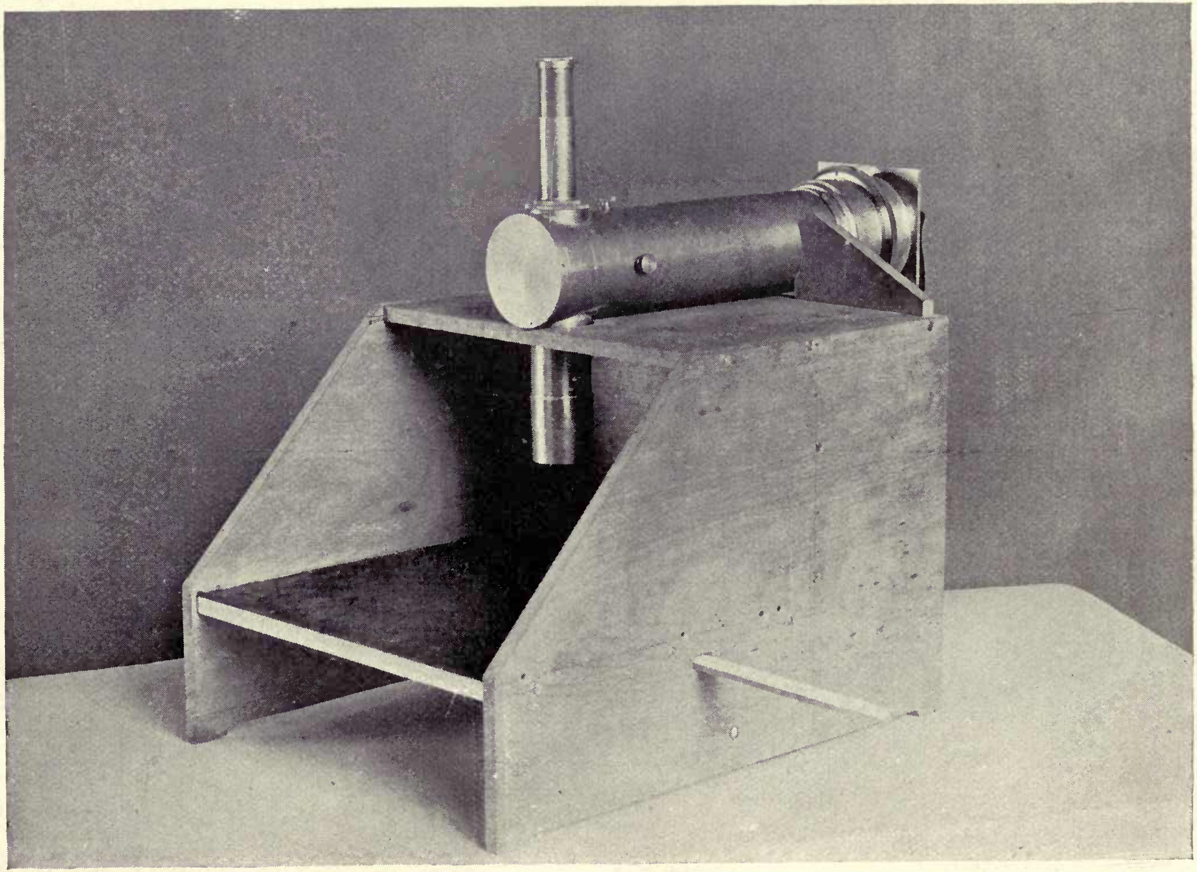

Fig. 221--PHOTOMETER.

122. Enlargement.—The plates having been allowed to dry, the highest density of each is measured and written in the margin. Messrs. Hurter and Driffield's photometer is a very perfect and precise instrument, but presents several disadvantages. It must be used in a dark room, two powerful lamps have to be set up and attended to, and the deposit of which the density is to be measured must have an area of at least one-quarter of an inch in diameter, a condition seldom found in landscape negatives. It has been found advisable to adopt here, another form of photometer shown in Fig. 221 ; a section is given in

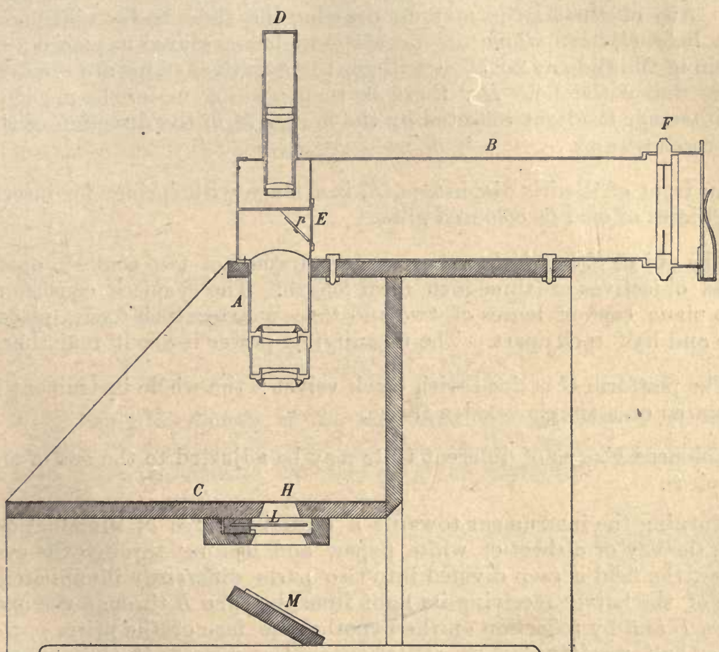

Fig. 222

Fig. 222. Although not by any means an accurate instrument, the densities measured by it are quite precise enough for the calculation of exposures in enlarging.

It consists of a microscope *A*, of very low power, mounted over a platform *C* on which rests the negative to be measured. The film being in contact with the platform, its image is formed in the horizontal plane passing through the axis of a large tube *B* closed at one end by

an iris diaphragm *F*. In *p* is a rectangular prism mounted on a diaphragm, which closes the other end of the tube *B*, and immediately in front of the prism is a piece of opal glass *E*. The upper face of the prism is in the focal plane of the microscope, and its edge divides the field into two equal parts.

Under the platform *C* is a slide *L* carrying a photographic plate, on which five strips of different densities have been impressed by a series of exposures. These densities increase in arithmetical proportion, the highest density being about equal to the density of the opal glass *E*. Any of these strips may, by drawing the slide, be brought under the hole *H*, over which the negative to be measured is placed; by pushing the slide as far as it will go, the negative strips are removed from under the hole *H*; there is then nothing under the negative to intercept the light reflected by the mirror *M* in the direction of the microscope's axis.

In front of the iris diaphragm, *F*, is a stage with springs for inserting pieces of opal or coloured glass.

The object glass of the miscroscope is formed of two common opera glass objectives of three-inch focal length. The eyepiece consists of two plano convex lenses of two and three-quarter inch focus, placed one and half inch apart. The magnifying power is about four times.

The platform *C* is lined with black velvet; the whole instrument is mounted on a simple wooden frame.

Coloured glasses of different tints may be adjusted to the end of the eyepiece.

Turning the instrument towards a uniform source of illumination, like the sky or a sheet of white paper, and looking through the eyepiece, the field is seen divided into two parts differently illuminated; one of the halves receiving its light from the tube *B* through the opal glass *E* and by reflection on the hypothenuse face of the prism *p*, the other half receiving it by reflection on the mirror *M* through the object glass of the microscope. The illumination of each of the two halves of the field can be varied at will, the one belonging to the tube *B* by opening or closing the iris diaphragm, and the one belonging to the microscope by moving the slide *L*. A negative being placed over the hole *H*, equality of illumination can be obtained by a judicious use of the iris diaphragm and of the slide.

The principle of this instrument is, that the illumination of the opal glass *E* is proportional to the area of the opening in the iris diaphragm. Without the opal glass, the opening or closing of the diaphragm does not cause any change in the illumination of the prism, so long as the

section of the bundle of rays coming out from the eyepiece is larger than the pupil. When a diffusing screen, like opal glass, is inserted, the light reaching the eye is composed of two parts: the light diffused by the screen, which is proportional to its illumination or to the opening of the diaphragm, and the light transmitted directly which is not affected by the opening of the diaphragm. With opal glass of density 2, this last part of the light is not appreciable and the illumination varies, within the limits of accuracy of the instrument, proportionally to the aperture of the diaphragm.

Let us assume, that the density D of the darkest strip of the slide L is such that, by placing it under the hole H, the illumination I of the two halves of the field is equal when the diaphragm is open to its full diameter a. Now, place a negative over the hole H, and close the diaphragm until equality of illumination is restored: let a' be its diameter and I' the illumination; we have:

$$\frac{I}{I'} = \frac{a^2}{a'^2}$$

But $\frac{I}{I'}$ is the opacity of the negative, and, as the density is the logarithm of the opacity,

$$\triangle = 2(\log a - \log a')$$

$\triangle$ being the density of the negative. The graduation of the diaphragm is calculated by this formula, the full opening being marked zero. The divisions of the scale are given hereunder:

Division of the scale.	Diameter of diaphragm's aperture.
0·0	1·00
·1	·89
·2	·79
·3	·71
·4	·63
·5	·56
·6	·50
·7	·45
·8	·40
·9	·35
1·0	·32

We are now able to measure any density greater than D and less than $D+1$, D being the density which just secures equality of illumination when the diaphragm is fully open or when the index is at zero

of the scale. The negative is placed over the hole H, the density strips are removed from under it by pushing the slide, and the diaphragm is closed until the two halves of the field are equally bright. Let a be the division of the scale opposite the index: the density is

$$\triangle = D + a$$

When the density of the negative to be measured is less than D, one of the strips of the slide L is drawn under the hole H, so as to form with the negative a density greater than D. The diaphragm is again closed until the two halves of the field are equally illuminated. Let a be the corresponding division of the diaphragm's scale, and d the density of the slide's strip; the total density measured is $\triangle + d$, and we have:

$$\triangle + d = D + a$$

or:

$$\triangle = (D - d) + a.$$

We mark on the slide the value of $D - d$ for each of the five strips, and the instrument is ready for measuring any negative. The diaphragm is first fully opened, and the negative placed over the hole H. Looking through the eyepiece, the slide is drawn until the negative appears darker than the prism, and the diaphragm is closed until equality of illumination is obtained. The density is the sum of the numbers read on the slide and on the diaphragm's scale.

Generally, there is a difference in the colour of the two halves of the field; to adjust accurately the illumination, it is necessary to place on the eyepiece a glass of the complementary colour.

The values of $D - d$ to be inscribed on the slide, may be ascertained by using a negative of which the various densities have previously been measured in a Hurter and Driffield's photometer. They may also be found by measuring the densities of the slide, as follows:—Take a negative of density slightly greater than D, and place it over the hole H; push in the slide, and close the diaphragm until the illumination of the field is uniform. Let a be the reading of the diaphragm's scale. Then draw under the negative the first or lightest strip of the slide and let B be the reading of the diaphragm's scale. The density of the first strip is

$$d = B - a$$

The difference between this strip and the next one is found in the same way, and the operation is repeated until the density d' of the last strip

is obtained. Let γ be the reading of the diaphragm's scale when equality of illumination is produced with the last strip d', then:

$$d' = D + \gamma$$

from which D is readily obtained.

d' must be equal to or greater than D; if greater, the number corresponding to it, to be marked on the slide, is negative.

For very great densities, a screen reducing the light is inserted in front of the iris diaphragm; this, however, is seldom necessary.

In measuring a landscape negative, it is advisable to cover it with a sheet of black paper in which a small hole has been made. The negative is shifted on the platform until the spot to be measured is bisected by the edge of the prism; the black paper is then adjusted over the negative so as to cover everything except this spot.

A little discrimination has to be exercised in selecting the spot of highest density in a negative. We must bear in mind that all lesser densities will be represented in the print by a deposit. It may occur that one particular spot has a density much in excess of the other parts of the negative, in which case it is not correct to regulate the exposure by it. The density to be measured is the most transparent spot of the negative which it is desired to have represented by pure white in the print.

The measurement of densities, with the photometer just described, is a very rapid operation.

Before developing the plates, it is necessary to ascertain the range of gradation or the highest density required for enlargement. Messrs. Hurter and Driffield have shown how this is done for enlargements on transparency plates or bromide paper; their instructions should be carefully adhered to. But some simplifications may be introduced in the details of the operations; they are illustrated by a description of the method followed here for enlarging on bromide paper.

The range of the paper is ascertained, as recommended by Mr. Driffield, by a series of exposures progressing in geometrical ratio. If the revolving disc is not available, the gradated exposures may be given by covering successively different portions of the paper. The series may be commenced, for instance, by exposing for three seconds the whole of the paper strip, except a margin on which the white of the paper is to be preserved. Then, a small portion of the strip is covered and a further exposure of one second given; a little more of the strip is covered and another exposure of two seconds given, and so on. The

first portion has received an exposure of three seconds, the second portion four seconds and the third portion six seconds. The separate exposure may be :

$$3^s—1—2—3—4—6—9—14—21—31—47—70,$$

the total corresponding exposures are :—

$$3^s—4—6—9—13—19—28—42—63—94—141—211.$$

That is very nearly a geometrical progression. The exposures need not be made to the light of a candle ; indeed, it is preferable to use a lamp which has been burning for some time, the light being more steady. The distance of the lamp is adjusted so that the first exposures shall not produce any appreciable deposit on development.

A strip of bromide paper was exposed as described above: after full development, it was found that the first well-marked change in the whiteness of the paper corresponded to the exposure of 4 seconds, while after the exposure of 141 seconds, there was no perceptible increase in the intensity of the black. The range of the paper was, therefore, $\frac{141}{4}$ or 35. If the enlarging factor of the negatives were 1·4, their highest density, exclusive of fog, should be :

$$\frac{\log 35}{1 \cdot 4} = 1 \cdot 10.$$

To find the exact value of this highest density, a plate is given four or five exposures calculated to give densities, exclusive of fog, little different from the highest density 1·10, already found. In the example given, the densities may be 1·06, 1·08, 1·10, 1·12, 1·14. A few trials may have to be made before the correct densities are obtained.

This plate, which we will call the standard tint plate, is inserted in the slide of the enlarging lantern, the image is projected on a piece of bromide paper and a series of exposures is given by covering successively the paper across the images of the density strips. The mean exposure must be timed to just produce a faint deposit on the developed paper for the image of the middle density strip. Assuming this mean exposure to be 45 seconds, the series should be :

$$39^s,\ 42^s,\ 45^s,\ 48^s,\ 52^s.$$

The resulting image on the paper consists of rectangular figures of different tints, as shown by Fig. 223. The darkest tints are those corresponding to the clear glass and gelatine in the standard plate; if the exposures have been properly timed, one at least of the tints in this strip must be the darkest black of the paper. In the figure, this tint has been produced by the exposures of 48 and 52 seconds; 48 seconds is, therefore, the exposure for which the first appreciable tint should be produced through the highest density of the negative. An examination of the strip which has received the exposure of 48 seconds shows, that this first tint was produced through the density 1·08; we must, therefore, endeavour to so develop our negatives that the highest density of each, exclusive of fog, shall be as nearly as possible 1·08, because they will then correspond precisely to the range of the paper, an exposure of 48 seconds just producing black for the darkest shadows and the lightest tint of the paper for the high lights.

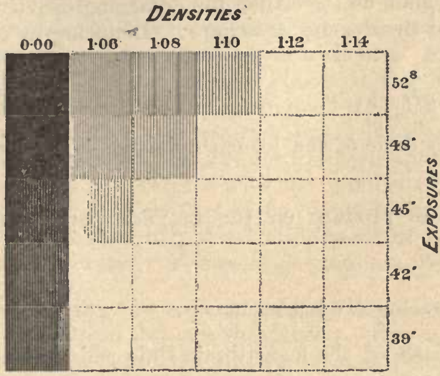

Fig. 223

To find the exposures for negatives of different densities, take a logarithmic slide and glue a piece of paper over the upper scale M (Fig. 224). On the edge of the paper, measure a distance AB equal to the logarithm of the range of the bromide paper, 35 in the example: this distance AB represents the density found by the experiment with the standard tint plate, 1·08 in our case, and is to be divided into equal parts, the point B being at 1·08 of the scale. Slide this scale until B coincides with number 48 of the logarithmic scale, which number is the exposure found with the standard tint plate; the slide is now set to indicate the exposure for a negative of any density, this exposure being the number of the logarithmic scale which coincides with the density of the negative on the upper scale. For instance, a negative of density 0·90 requires an exposure of 27 seconds, and one of density 1·20 requires 71 seconds.

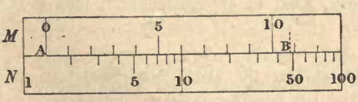

Fig. 224

The explanation is as follows: Let E and D be the exposure and density found with the standard tint plate, and r the range of the

bromide paper; since the exposure E has produced through the density D the same effect on the bromide paper as an exposure r times less through the clear glass and gelatine, the enlarging opacity is r and the enlarging value of the density D is log r. Denoting by a the enlarging factor,

$$aD = \log r.$$

By construction, the density scale of the logarithmic slide is a scale of the values of aD.

The exposure e necessary to produce on the bromide paper the lightest tint is $\frac{E}{r}$, hence:

$$\log e = \log E - \log r$$

e is the number of the lower scale of the logarithmic slide which coincides with zero of the density scale.

In order that another exposure E' through a density D' shall produce the same tint on the bromide paper, we must have:

$$\log E' = \log e + aD'.$$

This value of E' is, it will readily be seen, given by the lower scale of the logarithmic slide opposite the division D' of the upper scale.

The standard tint plate is made on a plate of the same batch of emulsion as those used on the survey, and is developed in the same manner.

It has been assumed that the fog densities of this plate and of all the negatives were equal. With plates of one emulsion, exposed and treated as they are here, this assumption is practically correct; with our plates the fog density never differs much from 0·15. As a matter of fact, this fog is never measured, all densities employed including fog: the only change which this necessitates, in the method outlined above, is a shifting of the zero of the density scale of the logarithmic slide. (Fig. 224.) The division formerly marked zero is now marked 0·15, the fog density, and the division 1·08 is marked 1·23; or, more simply, the density scale is moved 0·15 to the left.

In deciding which is the lightest tint indicating the commencement of the range of bromide paper, it is well to remember Mr. Driffield's advice and to disregard the faintest deposits. The change in the tints is so slow in this portion of the paper range that, if it were used, the different shades of the high lights in the negative would not be adequately rendered.

Although these operations may appear complicated, they are in reality very simple. Once the standard tint plate and the logarithmic slide have been prepared, they serve for the whole of one season's work. The only thing that remains to be done is to adjust the logarithmic slide whenever enlargements are to be made. For that purpose, a series of exposures is made with the standard tint plate, as already explained, but this time, the only portion of the print examined is the strip corresponding to the exposure through the clear glass and gelatine. The density scale *M* (Fig. 224) of the logarithmic slide is then shifted, until the point *B* coincides with the number of the other scale expressing the exposure which has first produced black on the print. The paper does not require to be fixed, and the whole operation is complete in three or four minutes. The enlargements can then be proceeded with.

In making this experiment, it will be observed that while the light tints produced through the density strips merge imperceptibly into the white of the paper, the gradations cease abruptly when black is attained; this method of setting the logarithmic scale is therefore very precise. From this peculiarity of bromide paper, two conclusions may be drawn: firstly, that it may be accurately timed, and secondly, that it must be accurately timed. Unlike dry plates, there is no latitude of exposure; the negative and the exposure must both be adapted to the paper. If, for some reason, the instructions given above cannot be followed, enlargements on bromide paper should not be attempted; they would result in failure. In such a case, transparency plates should be resorted to, because some use may be made of an imperfect transparency, while an imperfect bromide print is useless.*

The prints are developed with iron oxalate, washed in acidulated water and fixed. After a thorough washing, they are dried flat and in such a way that the expansion or contraction of the paper is equal in all directions. For this purpose, the prints may be soaked in alcohol before drying, or the edges of the paper may be held by pins or weights, where the contraction is too great.

Enlarging may be done either by daylight or by artificial light. Considering that the enlargements are all of the same size, the daylight apparatus may be extremely simple, but, it is essential that it should be made with precision; otherwise, the prints will be distorted. The plane of the negative and the plane of the paper must be parallel; the lens must be rectilinear. The apparatus should be inclined at an angle of 30° or 35°, and point to the northern part of the sky. The

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*We are now using a kind of bromide paper recently introduced, and called "Platino Bromide." It has a much longer range than ordinary bromide paper (about 150), and requires a density of 1·80 in the negatives. The black tones are matt, and give to the prints a more artistic appearance. A special kind made on paper of the thickness of a thin visiting card, is well adapted to survey photographs.

disadvantage of daylight is its inconstancy; when employed, its use should be restricted to days when the sky is quite clear, and then only to the middle of the day, when there is little change in the intensity of light.

Artificial illumination is more convenient and always available, but requires more elaborate apparatus. For enlargements of moderate amplitude, like those needed for surveying, the best source of light is probably a spiral electric lamp,* provided the pressure on the mains is uniform. While in use, other lights which may happen to be on the same circuit should not be turned off or on. It is important to so adjust the lamp, that the image of the spiral formed by the condenser shall be exactly in the centre of the diaphragm of the enlarging lens. The condenser not being achromatized, the image is strongly coloured on the edges. If focussed for the actinic rays, the fringe of colour must be red; when it is blue, the lamp is too far from the condenser.

The insertion of diaphragms does not reduce the illumination, so long as the aperture is larger than the image of the source of light; a lens of moderate aperture may therefore be employed with a spiral electric lamp. With other sources of light, the useful portion is that of which the image is inside of the aperture of the diaphragm; the other parts might as well be covered and, in fact, should be covered on the general principle of photography that any light which is not useful is prejudicial.

Slight distortions are caused by the play of the negative carrier in the lantern and by the bromide paper not lying quite flat on the copying board. To minimize this cause of error, it is essential to employ a lens of long focus.

There does not appear to be any special advantage to enlarge to one size rather than another. The proportion adopted here (about 2·1) was simply calculated to fill in the width of the bromide paper. Once fixed, however, the proportion should not be changed.

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*This lamp is made by the Edison and Swan United Electric Light Co., Ltd. They call it "Focus Lamp."

CHAPTER VII.

Field Work.

123. Triangulation.—The triangulation may be executed at the same time as the topographical survey, but it is preferable to have some of the principal points located in advance by a primary triangulation.

The subject is fully treated in the standard works on surveying; very little requires to be added here. There exists, however, some misconception as to the order to be followed in the operations: a few words of explanation may prove useful.

A survey must be considered as consisting of two distinct operations; one has for its object the representation of the shape or form of the ground, the other the determination of its absolute dimensions. A perfect plan or triangulation can be made without the measure of any base or length; the plan will exhibit the various features of the ground in their exact proportions, but no absolute dimension can be measured on it until the scale of the plan has been determined. This is done by measuring on the ground one of the dimensions represented on the plan: so, the object of the measure of a base is to fix the scale of the survey and nothing more.

To execute a triangulation, the surveyor is recommended to commence by measuring a base and making it the side of a triangle, on which he is to build other triangles of increasing dimensions. There is a certain logical sequence in the order followed, but in strict theory, the order is immaterial, the triangulation may be executed first and when completed, connected with a base by triangles decreasing in size as they come near the base.

In practice, the case is different: there are several advantages in executing the triangulation first.

The selection of a base is governed by various considerations: the ground must be tolerably level and free of obstacles, and the direction, length, and position of the base must be such as to permit a good connection, by triangles of proper shape, with the main triangulation. The surveyor can make a better choice after he has been over the

whole ground, than on his arrival when he has seen little of it. Having established the main triangles, he also knows best how to connect them with a base. In a mountainous country, the principal summits of the triangulation are fixed by nature and cannot be changed, while the position or direction of the base may generally be modified to some extent. Were the base measured first, it might be found not to connect properly with the main triangles.

The secondary triangulation is the work of the topographer, and the construction of signals on the secondary points should be his first act upon arriving on the ground.

Should the time at his disposal allow, he will not commence the survey proper until all signals have been established, otherwise he may have to measure angles between points not very well defined. When he does so, the closing error of a triangle is assumed to be due to the want of definition of the points.

Let A, B and C represent the angles of a triangle whose summits have been occupied in the order given. At A, the surveyor observes the angle between B and C, where there are no signals. He puts up a signal at A and moves to B. In measuring the angle between A and C, he has a signal at A and none at C. Placing a signal at B, he measures the third angle C between two signals.

Call a the closing error of the triangle and ε the probable error of a sight on a point without signal. The probable errors of the angles are :—

For A, $\varepsilon\sqrt{2}$

" B, ε

" C, O.

The corrections to the angles must be proportional to the probable error of each ; they are :—

For A, $\dfrac{a}{1+\sqrt{\frac{1}{2}}}$

" B, $\dfrac{a}{1+\sqrt{2}}$

" C, O.

The closing error must not exceed a certain limit fixed by the degree of precision of the survey : when the limit is exceeded, the stations must be re-occupied, commencing at the most doubtful one.

The stations of the primary triangulation are the last ones to be occupied when they have been established by a previous survey.

To have a correct idea of the work he is doing, the surveyor must make in the field a rough plot of his triangulation, on which he marks all the stations occupied. It shows him the weak points of the survey and enables him to plan his operations with more assurance.

The object of the secondary triangulation is to fix the camera stations: its summits are selected for that purpose only. All the topographical details of the plan are drawn from the camera stations.

124. Camera stations.—A camera station is fixed either by angles taken from the station on the triangulation points, or by angles taken from the latter, or by both. It is easier and more accurate to plot a station by means of angles taken from the triangulation points than by the angles measured at the station; therefore, the camera stations should, if possible, be occupied before the triangulation summits. There are, however, other considerations which may interfere.

Camera stations must be chosen in view of the construction of the plan by the method of intersections: other methods are to be employed only, when this one fails or when the data collected on the ground are not sufficient to furnish enough intersections.

A conspicuous mark or signal of some kind should be left at each station; it does not require to be very elaborate, a pole or a few stones would be sufficient. Angles on this signal are measured from the triangulation points, in order to place the station on the plan.

It seldom occurs that the camera is set up precisely at a triangulation point. Generally, it is advisable to move a few feet in one direction or another, to include in the view a certain part of the landscape. Whenever there is any advantage in displacing the camera, the surveyor should not hesitate to do so. The distance from the triangulation point measured with a light tape and an angle read on the instrument, locate the camera station.

For the same reason, it is not necessary that several views be taken from each station: every view should be taken from the point where it is best for the construction of the plan. The greater number of stations gives very little extra work, either in taking the angles for fixing their positions, or in plotting them.

In general, views taken from a great altitude and overlooking the country are desirable, but there are numerous exceptions. Those taken from low altitudes are of great assistance in drawing the contour lines of the valleys.

Sometimes difficulties may exist in obtaining two views which will furnish intersections over a certain part of the ground. In such a case, the method of vertical intersections may be employed, views being taken from different altitudes. Provided the difference of altitude is large enough and the points to be determined not too far, the precision is the same as with horizontal intersections.

It would be desirable to have the views of the same part of the ground taken at the same time of day; the shadows cast being identical, it is more easy to recognize the different points. It would be well, also, to avoid views taken looking towards the sun; they are flat and lack detail. But, the surveyor has other considerations to take into account; he is seldom able to choose his own time for occupying a station or taking a photograph. He has often to take views against the sun or dispense with them altogether. With care in cutting off the sky, he may still obtain results remarkably good under the circumstances.

The identification of points, even under different lighting, does not offer any serious difficulties. The number of photographs must be sufficiently large to cover the ground completely: an additional view causes very little extra work, either in printing or plotting, and may save much trouble. The surveyor should not hesitate to take one whenever he finds a place where it may be useful.

Two or three points in each view are observed with the altazimuth, the altitudes and horizontal angles between them being noted. The altitudes serve to check the horizon line on the photograph, in case the camera should be slightly out of level, and the horizontal angles are for the orientation of the view.

The notes of observations are kept in the usual manner for such work, the points observed upon being shown on sketches made on the spot. The sketches serve to identify points with more certainty than a mere designation by a letter or figure.

CHAPTER VIII.

PLOTTING THE SURVEY.

125. SCALE OF PLAN.—The minutes of the Canadian Surveys are plotted on a scale of $\frac{1}{20000}$; they are afterwards reduced for publication to $\frac{1}{40000}$. The equidistance is 100 feet.

The convention already adopted in perspective (§ 54) must be recalled here; the angles measured and the photographs taken are conceived to have been measured and taken on a model of the ground already reduced to scale.

That the perspectives obtained from any point of such a model are identical with those taken from the similar point of the ground has already been shown (§ 54); the same rule applies to photographs, in theory at least.

The angles measured are also equal to those of the ground, for any triangle ABC (Fig. 225), of the ground is represented on the model by a similar triangle abc. The altazimuth set in a gives between b and c the same angle as it would between B and C, if set at A.

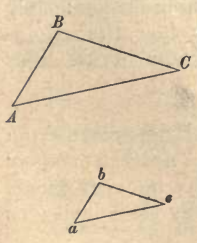

Fig. 225

Thus, if the plan be required on a scale of $\frac{1}{20000}$, the model is assumed to have been reduced to that scale, and the problem consists in making a plan full size by means of angles and photographs obtained on the model.

No change being made to the camera, the focal length preserves the same value; if one foot, it covers on the model a distance corresponding to twenty thousand feet on the ground.

The plan and the model being both reduced to the scale of $\frac{1}{20000}$, it is clear that, if this scale be used to measure an actual dimension on either, the result is the number expressing the corresponding actual dimension on the ground. If a division of the scale be called a "scale foot," a dimension of the ground is expressed in real feet by the same number which expresses in "scale feet" the corresponding dimension

of the model or plan. A distance of a mile contains 5280 real feet on the ground, and is represented on the model by 5280 "scale feet."

The focal length of one foot mentioned above would be a focal length of 20,000 "scale feet."

It follows that, although the problem consists in representing a model full size, the scale may be employed to measure the actual dimensions, the value of one division being considered as an arbitrary unit.

In other words, a Liliputian surveyor must be imagined to be operating in a Liliputian country of which he wants to make a plan full size. His camera is of enormous dimensions, bearing to him the same proportion as a camera several miles long would to an ordinary man.

Of course, all the constructions used in plotting the plan can be demonstrated without such an hypothesis, but the explanations would not be so simple, and it would not be so easy to grasp the whole subject.

126. Plotting the triangulation.—The primary triangulation is assumed to have been previously calculated; the primary stations can therefore be plotted at once by their co-ordinates.

The angles of the secondary triangles are now calculated, and the corrections indicated by the closing errors applied. Some of these triangles have common sides with the primary triangulation: they are calculated first. With the values found for their sides, the adjoining triangles are calculated, and so on, until the lengths of all sides have been obtained.

With these values, the differences of latitude and departure from every summit of the secondary triangulation to the nearest primary station are calculated. Unless the primary triangles be very large, the secondary stations can be plotted on the plan by their latitudes and departures without any appreciable error.

The camera stations are next placed by the angles observed upon them from the triangulation points. These angles are plotted with a vernier protractor, or by means of a table of chords; either method is accurate enough for the purpose.

As long as a sufficient number of readings have been taken on a camera station from triangulation points, no difficulty is experienced in placing the station; it is not so when only a limited number of readings, or none at all, are available. There are two cases to consider.

Case I.—The camera station has been observed from one or more triangulation points. The camera station M (Fig. 226) having been observed from the triangulation point A, triangles may be formed with M, A and other triangulation points observed both from A and M, such as B. In the triangle MAB the angles at M and A have been observed, and

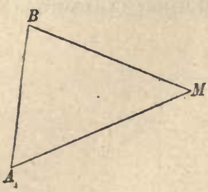

Fig. 226

$$B = 180° - (A + M).$$

Similar calculations being made for other triangulation points give the directions of the station as seen from these points; the plotting is done as if the station had been observed from every such point.

Case II.—The camera station has not been observed from any triangulation point. In this case the station must be placed by the angles which have been observed from it. This can be done either by describing, through the points observed, circles containing the angles between them, or by the use of a station pointer. The first method requires complicated constructions and is not very accurate, and the station pointer can serve only for three points at a time. The following process will be found rapid and accurate when many points have been observed from the station:

On a piece of tracing paper, take a point to represent the camera station, and draw the directions of all the points observed. Put the tracing paper upon the plan and try to bring every one of the directions drawn to pass through the corresponding point of the plan. The camera station is then in its place.

From the foregoing it is evident that the surveyor should endeavour to obtain at least one direction from a triangulation point on every camera station: the plotting is less laborious and the result more accurate.

The use of photographs for placing camera stations must be avoided; the precision is not sufficient.

127. Checking the photographs.—Before making any use of the photographs, it must be ascertained that they have not been distorted in the operation of enlarging. Join the middle notches HH', PP', Fig. 227, and with a set square test these two lines for perpendicularity. Take with a pair of compasses the distance of the two notches A and B, which is one-half of the enlarged focal length, and see whether it is equal to the distance of the two notches C and D. Then apply one of the points of the compasses in P; the other point must come in

E and F. Transfer the point to P' and check $P'G$ and $P'J$. If the photograph stands all these tests, it may be depended upon as accurate;

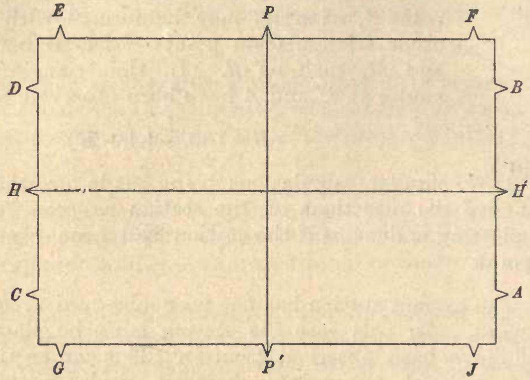

Fig. 227

if it does not, it is returned to the photographer with a request for a better one.

128. Plotting the traces of the picture and principal planes.—The traces of the picture and principal planes are now drawn on the plan. Every photograph contains at least one and generally several of the points marked on the plan. Find the distance Sa, Fig. 228, from the station to the projection of such a point a of the photograph on the horizon line; PS is taken on the principal line equal to the focal length and Pa equal to aa'. The whole of this construction is made on the "photograph board" which will be mentioned further on.

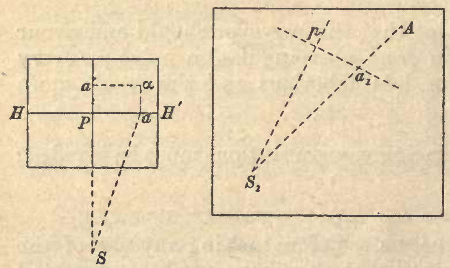

Fig. 228

On the line S_1A of the plan representing the direction of a, take from the station S_1 the distance S_1a_1 equal to Sa; from a_1 as centre with aa' as radius, describe an arc of circle and draw S_1p tangent to it: it is the trace of the principal plane. The trace of the picture plane is the perpendicular to S_1p passing through a_1.

Instead of making the construction on the photograph board, it can be made on the plan. On S_1A take S_1B (Fig. 229), equal to the focal length, erect BC perpendicular to S_1A and equal to aa' (Fig. 228). Join S_1C and take S_1p equal to the focal length: at p erect a perpendicular to S_1C; it is the trace of the picture plane, and S_1C is the trace of the principal plane.

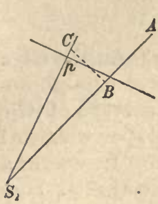

Fig. 229

The first method is preferable, because it does not require so many construction lines on the plan.

The trace of the principal plane is marked only where it intersects the picture trace so as not to confuse the plan.

129. Identification of points.—The survey being plotted mainly by intersections, it is necessary, after selecting the points on one photograph, to identify these same points on another photograph covering the same ground. The points are chosen on those lines which best define the surface, such as ridges, ravines, streams, crests, changes of slope, etc. Each point is marked on the photographs by a dot and a number in red ink.

Ability to identify points is acquired by practice; it is surprising to see with what rapidity and certainty a surveyor familiar with the work can not only pick out the points on several photographs, but also find as many as he wants. Beginners may, in case of difficulty, resort to Prof. Hauck's construction (§ 86). The two photographs are pinned, side by side, on a drawing board. The images of the stations, if they appear on the photographs, are the *kern* points; if outside of the pictures, they are plotted from the plan on the drawing board. The parallels to the principal lines, on which the scales are to be placed, are drawn as explained in § 86, and the scales fixed in position. A fine needle is fixed at each of the *kern* points; to it is tied a fine silk thread, the other end of which is fastened to a light paper weight by a fine rubber band. A well defined point is identified on the two photographs; its image must be far enough from the *kern* points. Taking one of the paper weights in one hand, the silk thread is given enough tension to keep it straight; it is displaced to pass through the point which has just been identified and the weight is deposited on the drawing board, still keeping the tension. This operation is repeated with the other silk thread and the other photograph. The two threads should intersect the scales at the same division; if they do not, one of the scales is moved until the divisions intersected are identical. The identification of points may now be proceeded with. Having selected a point on one of the photographs, the silk thread is displaced to pass through the point and the

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intersection of the scale by the thread is noted. The other thread is now moved to intersect the same division on the other scale; the point looked for is under the thread.

130. Plotting the intersections.—The surveyor marks on the edge of a band of paper the distance from each point of one of the photographs to the principal line, and adjusts the paper on the trace of the picture plane previously drawn on the plan, holding it by paper weights: he repeats the same operation for the other photograph. Inserting a fine needle at each station, he fastens to it a black silk thread connected at the other end by a fine rubber band to a small paper weight. Holding the weight in one hand, he moves the thread until it coincides with one of the marks on the edge of the band of paper and he deposits the weight on the plan, giving sufficient tension to the rubber to keep the thread taut. Doing the same thing at the other station, the intersection of the two threads indicates the position on the plan of the point of the photographs.

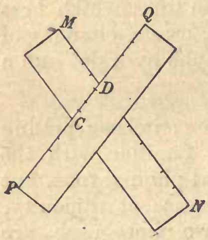


Fig. 230

When the bands of paper overlap, as in Fig. 230, the portion CD of the picture trace PQ is marked on the band MN which is under; the band PQ is placed in proper position and the marks on its edge transferred to the line CD. The band PQ is now placed under MN, the marks on the latter along CD serving the same purpose as those of PQ.

The station may be too close to the edge of the plan to plot the trace of the picture plane, as in A, Fig. 231, the picture trace falling in QR, outside of the plan.

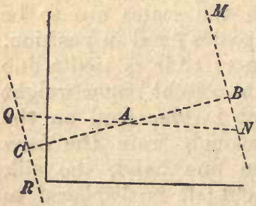


Fig. 231

In this case the trace AC of the principal plane is produced to B, a distance equal to the focal length, and MN is drawn perpendicular to BC or parallel to QR. The line MN occupies with reference to QR, the same position as the focal plane of the camera does to the picture plane of the perspective. The direction of a point of the photograph projected in Q on the picture trace, is found by joining NA and producing to the opposite side of A.

The first two intersections should be checked either by a third one or otherwise. They may, for instance, be checked by determining the height of the point from the two photographs: unless correctly plotted, the two heights obtained will not agree. This check, however, does not indicate slight errors.

The check may also be a line drawn by means of the perspectograph or perspectometer and on which the point is situated, such as the shore of a lake or of a river, but the best check is a third intersection.

The number of every point is inscribed in pencil on the plan.

131. PLOTTING WITH THE PERSPECTOGRAPH. To draw with the perspectograph the plan of a figure which appears on a photograph, the figure must be beyond the picture plane (§102) or above the ground line on the photograph. Thus the lake AB (Fig. 232), being below the ground line XY of the photograph, cannot be drawn without such a change of ground plane, as will bring the new ground line $X'Y'$ below the lake AB. It has been explained that this is done by doubling the height of the station until the ground line is brought into correct position (§102).

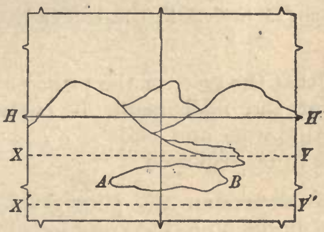


Fig. 232

The slide XY of the perspectograph, Fig. 233, is, by means of the scales drawn in X and Y on the drawing board, adjusted to a distance from RT equal to the focal length.

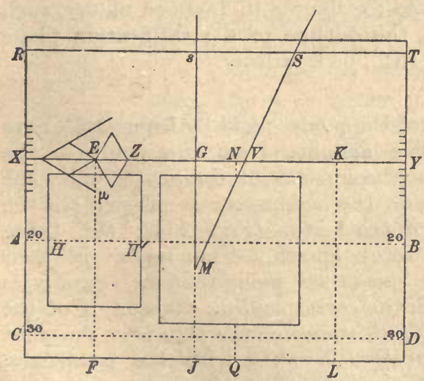


Fig. 233

After adjusting S, the pencil is brought over a point M of the trace GJ of the principal plane at a distance sM from s equal to twice the focal length.

The photograph is pinned under the tracer, the horizon line HH' over the corresponding line AB of the board and the principal line over EF: the iron rod connecting V and Z is then adjusted so as to bring the tracer μ midway between the horizon and ground lines.

The cross section paper is pinned to the board, one of its lines coinciding with the trace of the principal plane GJ, and other lines with the front lines AB and CD, drawn at known distances from the foot of the station s.

There will be no difficulty in tracing with the point μ the part of the photograph which, on the figure, is on the right of the principal line, but it may happen that in moving μ to the left, the obliquity of the arm MS would prevent the free play of the instrument. It should then be reversed, the slide XY being changed end for end, the photograph transferred from EF to KL, the cross section paper moved so as to bring, on the trace NQ of the principal plane, the line of the paper which was formerly over GJ, and the point S placed to the left of s; μ being now between the two slides RT and XY, the tracer has to be changed to the opposite arm.

The perspectograph can be so adjusted that the trace of the principal plane is the same in both positions of the instrument, it being sufficient not to move s, when inverting the arms and slide XY: the cross section paper then does not require to be displaced.

Having obtained the plan of the figure shown on the photograph, the reduction to the proper scale is made at sight on the cross section paper, and transferred to the general plan. The transfer should be checked by points previously established by intersections.

The use of the instrument is possible every time the plane of the figure can be determined, as for instance a lake, a river, a contour line, or the foot of a mountain. Slight differences of level do not affect the result when the height of the station is great.

The instrument can also be used for figures in inclined planes, such as a river with a rapid slope, the outline of a stratification plane which has not been distorted, a road, or a railway.

132. HEIGHTS.—The heights of the points fixed by intersections are found as explained in § 85. The distance from the point to the horizon line is taken with a pair of compasses on the photograph; one of the points of the compasses is placed on the division A of the sector (Fig. 234), OA being equal to the focal length. The sector is then opened until the other point of the compasses coincides with the corresponding division B of the other arm. The distance on the plan from the point to the picture line is taken with the compasses, then one leg being placed in A on the sector, the other will come somewhere in C, the compasses are then turned around on C and brought on the division D of the other arm corresponding to C. The line CD is the height of the point above or below the horizon plane, which means the height above or below the station.

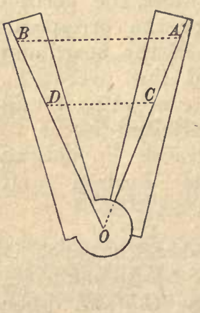


Fig. 234

Another method consists in making use of the angular scale shown in Fig. 235. Take SP equal to the focal length; erect the perpendicular PA to SP and divide both into equal parts. Join to S the points of division of PA and through those of SP draw parallels to PA.

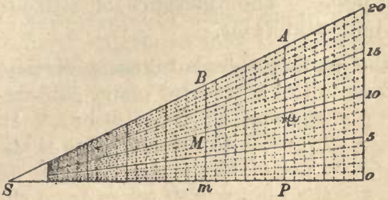


Fig. 235

Now, with a pair of compasses, take on the photograph the distance from the point of the perspective to the horizon line: transfer it to $P\mu$ and suppose that it is found to correspond to the line $S\mu$ passing through the point 9 of the graduation of PA.

Take with the compasses the distance on the plan from the horizontal projection of the point to the picture line, and transfer it to the right or left of P according as the point of the plan is beyond or within the picture line. Then take with the compasses the distance on a parallel mB to PA, between m and the point M where the line mB is intersected by $S\mu$, corresponding to 9 of the graduation. This distance mM is the height of the point above or below the station.

A scale is now pinned somewhere, perpendicularly to a line AB (Fig. 236), the division C of the scale corresponding to AB being the height of the station. The compasses are taken off the sector, and one of the legs being set in C, the other leg coincides with a division D of the scale, above or below C, which is the height of the point above the datum plane. This height is entered in pencil on the plan, enclosed in a circle, to distinguish it from the number of the station. It is checked by a second photograph, and, when the discrepancy between the two heights is within the limits of error admissible, the mean is entered in red ink on the plan and the pencil figures erased.

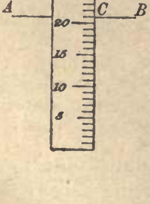


Fig. 236

A difference in the heights obtained from the two photographs indicates that the two points identified do not represent the same point of the ground, or that an error has been made either in plotting it, or in finding its height.

A third intersection disposes of the first two alternatives, and a new measurement of the height shows whether any error has been made.

133. Vertical intersections.—In the method of horizontal intersections the base line is projected on the horizontal plane; in this method it is projected on a vertical plane. The difference of altitude of the two stations must therefore be considerable.

The principal plane of one of the photographs is taken as vertical plane of projection; the ground plane is the horizontal plane containing one of the stations. In Fig. 237, the ground line is the trace of the principal plane of the photograph taken from the station A; the ground plane is the horizontal plane of the station B. On the ground plan, a and B are the two stations, CD and EF their picture traces. The station A on the vertical plane is on the perpendicular aA to XY equal to the height of A above B. A point such as p plotted by the method of horizontal intersections, would not be accurately fixed because the angle of the directions aD and BF is too small.

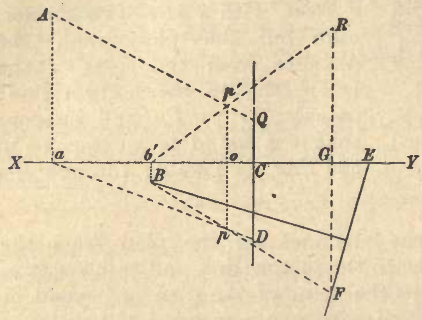


Fig. 237.

Project the visual rays from A and B on the vertical plane: the visual ray from A is a line AQ passing through the projection Q of the point's image on the principal line. It is drawn by taking CQ equal to the height of the point on the photograph above the ground line, and joining AQ.

The vertical projection of the visual ray from B is a line $b'R$ passing through the vertical projections of the station b' and of the point's image R, on the second photograph. To find R, let fall FG perpendicular to XY and produce to R, GR being equal to the height, on the photograph, of the point's image above the horizon line.

The intersection of AQ and $b'R$ is the vertical projection p' of the point. Letting fall the perpendicular $p'o$ to XY and producing, determines the position p of the point on the ground plan.

The construction gives not only the point on the ground plan but also its height op'. This process is the best one for plotting a narrow valley between two high walls: it has, however, the disadvantage of requiring a complicated construction.

134. Photograph board.—So many construction lines are employed on the photographs that it is advisable to have a photograph board on which part of the lines are drawn beforehand, once for all.

It consists of an ordinary drawing board, covered with strong drawing paper. Two lines at right angles, DD' and SS', Fig. 238, represent the horizon and principal lines; PD, PD' PS and PS' are each equal to the focal length, so that D, D', S and S' are the left, right, lower and upper distance points respectively.

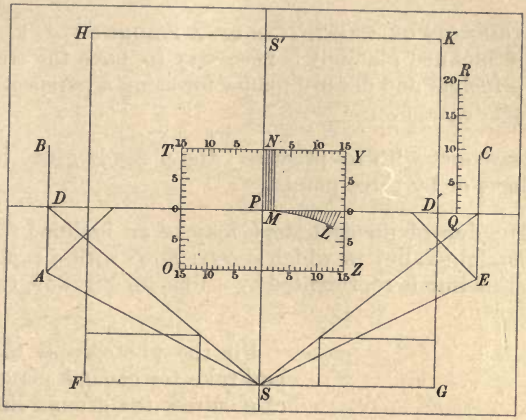


Fig. 238

The photograph is pinned in the centre of the board, the principal line coinciding with SS' and the horizon line with DD'. Four scales, forming the sides of a square $OTYZ$, are drawn in the centre, the side of the square being a little larger than the length of a photograph.

They answer various purposes as, for instance, drawing parallels to the horizon or principal lines by laying a straight edge on the corresponding divisions of the scales or marking the ground line by joining the divisions of the vertical scales representing the height of the station.

At a suitable distance from the distance point D' a perpendicular QR is drawn on which are marked, by means of a table of tangents, the angles formed with DQ by lines drawn from D. This scale is employed for measuring the altitudes or azimuthal angles of points of the photograph, as will be explained later on (§137). From S as a centre with SP as radius, an arc of circle PL is described and divided into equal parts. Through the points of division, and between PL and PD', lines are drawn converging to S. Parallels MN to the principal line are also drawn sufficiently close together. All these lines are used in connection with the scale of degrees and minutes QR.

The studs of the centrolineads are fixed in A, B, C and E; the lines AB and CE, joining their centres, and those required for adjusting the centrolineads, are drawn and used as explained in §97.

A square $FGKH$ is constructed on the four distance points.

135. Construction of the traces of a figure's plane.—When a figure is in an inclined plane, it is necessary to have the traces of the plane on the principal and picture planes for using a perspective instrument on the photograph.

Two cases are met with in practice: the plane is given by the line of greatest slope, or by three points.

Case I.—The line of greatest slope may be an inclined road, or the middle of a straight valley in which a river flows with a rapid current. On the plan, this line is represented by a line ab, Fig. 239, the altitude of a being known.

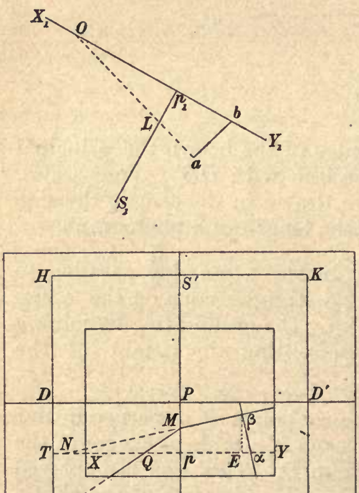


Fig. 239

Pin the photograph to the board and take for ground plane the plane of a: draw the ground line XY.

On the plan draw aO perpendicular to ab and produce it until it intersects the principal line S_1p_1 and picture trace X_1Y_1.

On the photograph take pE equal to p_1b; at E erect a perpendicular to XY and produce it to the intersection β with the perspective of the line of greatest slope. Take pN equal to p_1O and join $N\beta$: it is the trace of the required plane on the picture plane.

Take pQ equal to p_1L and join MQ: it is the trace of the required plane on the principal plane, supposed to be revolved around SS' on the picture plane, the station falling in D. Produce MQ to R: DR is the vertical distance of the station above the plane $RM\beta$. The new horizon and ground lines are now drawn as in §82·

Case II.—Take for ground plane the plane containing one of the points, a (Fig. 240), and draw the ground line XY on the photograph.

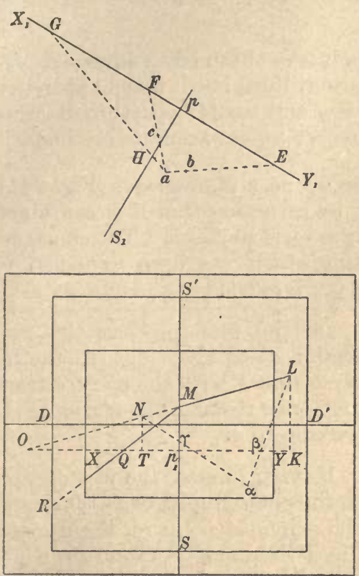


Fig. 240

Join a on the plan to the two remaining points and produce to the intersections E and F with the picture trace.

Take on the photograph p_1K equal to pE and draw KL perpendicular to XY; join the perspectives α and β of the points shown in a and b on the plan and produce to the intersection with KL. Take p_1T equal to pF, draw TN perpendicular to XY and produce to the intersection N with the line joining the perspectives α and γ. Join NL: it is the trace of the required plane on the picture plane.

Produce LN to O and take pG equal to p_1O; join aG and take p_1Q equal to pH. The line MQ is the trace of the required plane on the principal plane supposed to be revolved around SS' on the picture plane, the station being in D. Here also, DR is the vertical height of the station above the plane of the three given points. The new horizon and ground lines are constructed as previously explained.

136. Contour lines.—A sufficient number of heights having been determined, the contour lines are drawn by estimation between the points established. In a rolling country, a limi'ed number of points is sufficient to draw the contour lines with precision, but in a rocky region the infl-ctions of the surface are so abrupt and frequent that it is utterly impossible to plot enough points to represent the surface accurately. The photographs are of great assistance to the draughtsman; having them under his eye, he is able to modify his curves so as to represent the least inequalities of the ground.

Instead of drawing the contour lines at once on the plan, the draughtsman may commence by sketching them on the photograph in the same way as he would on the plan. Every point plotted has been marked on the photograph, and the altitudes may be taken from the plan. By adopting this course, he is able to follow very closely the inequalities

of the surface. The curves serve to guide the draughtsman in drawing those of the plan, or they may be transferred by the perspectograph or the perspectometer.

As long as a sufficient number of points is obtained by intersections, there is no difficulty in drawing the contour lines, but it may happen in a rapid survey, that the points are too few and too far apart for defining the surface. It is then necessary to resort to less accurate methods.

A mountain ridge, which appears in $\alpha\beta$ on a photograph (Fig. 241), can be divided by the contour planes, by assuming that it is contained in a vertical plane. The construction, which has been explaned in § 62, is carried out as follows:—

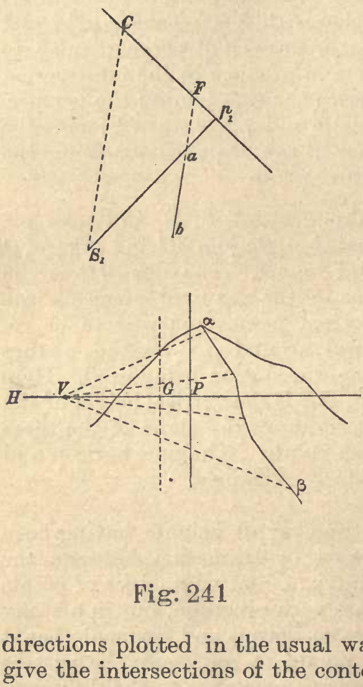


Fig. 241

On the plan produce the projection ab of the ridge, to the intersection F with the picture trace and draw through the station S_1C parallel to ab.

Having pinned the photograph to the photograph board, take from the principal point on the horizon line PV equal to p_1C and PG equal to p_1F. At G, place the scale of equidistances perpendicular to the horizon line, the division G corresponding to the height of the station, and join the marks of the scale to the vanishing point V.

Having now the points of intersection of the ridge by the contour planes, their distances from the principal line are marked on the edge of a band of paper and their directions plotted in the usual way. These directions produced to ab give the intersections of the contour lines.

When the mountain has rounded forms and no well-defined ridge, the visible outline must be assumed to be contained in a vertical plane perpendicular to the direction of the middle of the ridge. The construction is made by drawing, on the photograph board, SV perpendicular to the direction SM of the middle of the outline. (Fig. 242.) On the plan, p_1M_1 is taken equal to PM, and from the projection a of the summit of the mountain a perpendicular ab is let fall on S_1M_1 which represents the projection of the visible outline; it is

produced to the intersection N with the picture trace, PQ is taken equal to p_1N, and the scale of equidistance placed at Q perpendicular to the horizon line. The points of division are joined to V, produced to $\alpha\beta$, and the plotting done as in the preceding case; or the directions of the intersections of $\alpha\beta$ by the contour planes may simply be plotted and the contour lines drawn tangent to these directions.

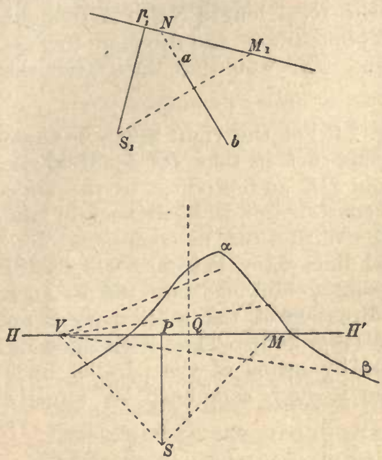


Fig. 242

The horizon line contains the perspectives of all the points at the height of the station; it is the perspective of a contour line when the height is that of a contour plane.

Full details on the plotting of contour lines being given in the text books on surveying, it is not necessary to repeat them here. The main point is to understand thoroughly the mode of formation of the surface and its variations under different circumstances; the surveyor should pay particular attention to the subject, making a special study of it. Without this knowledge, the proper representation of the ground would require the plotting of a very large number of points.

137. Photograph protractor.—The angle between the point of a photograph and the principal and horizon lines, that is, the altitude or azimuthal angle, is sometimes wanted.

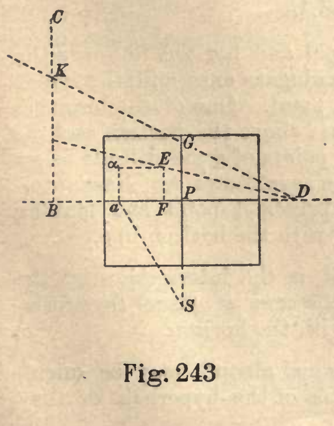


Fig. 243

The azimuthal angle is obtained at once on the photograph board by joining the station S (Fig. 243) to the projection a of the point on the horizon line. If required in degrees and minutes, the distance Pa is transferred to the principal line in PG; D is joined to G and produced to the scale of degrees and minutes BC, where the graduation K indicates the value of the azimuthal angle.

Were many such angles to be measured, the horizontal scales TY and OZ

(Fig. 238) might be divided into degrees and minutes by means of a table of tangents, using as radius the focal length *SM*. A straight edge being placed on a point of the photograph, and directed to pass through identical divisions of *TY* and *OZ*, would at once give the azimuthal angle of the point.

The altitude is the angle *S* (Fig. 243) of the right angle triangle having for sides *Sa* and *aa*. To construct it, take *DF* equal to *Sa*, draw *FE* parallel and equal to *aa*, join *DE* and produce to the scale of degrees and minutes *BC*. This construction is facilitated by the lines previously drawn on the board. With a pair of compasses, take the distance from *a* to the principal line, carry it from *P* (Fig. 238) in the direction *PD′*, and from the point so obtained take the distance to the arc *ML*, measuring it in the direction of the radii marked on the board: this is the distance *PF* (Fig. 243). Then with the compasses, carry *aa* to *FE*, which is done by means of the parallel lines *MN* of Fig. 238. The construction is now completed as already explained.

A protractor may be constructed to measure these angles. It consists of a plate of transparent material on which are lines parallel to the principal line, containing the points of same azimuth, and curves of the points of same altitude.

The azimuthal lines are constructed by plotting the angles in *S* and drawing parallels to the principal line through the points of intersection with the horizon line.

Denoting by *h* the altitude of a point *a* and taking the horizon and principal lines as axes of co-ordinates, the equation of the curve of altitude *h* is :—

$$y^2 = (x^2 + f^2) \tan.^2 h.$$

This is an hyperbola of which the principal and horizon lines are the transverse and conjugate axes and the centre is the principal point. One of the branches contains the points above the horizon and the other branch the points of same altitude below the horizon. The asymptotes are lines intersecting at the principal point and making angles equal to *h* with the horizon line.

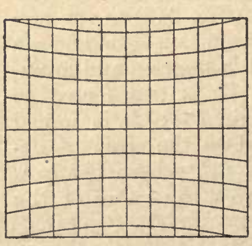


Fig. 244

This hyperbola is the intersection by the picture plane of the cone of visual rays forming the angle *h* with the horizon.

The curves of equal altitude may be calculated by the formula of the hyperbola or they

may be plotted by points, reversing the construction given above for finding the altitude of a (Fig. 243). The complete protractor is shown in Fig. 244: the angular distance between the lines depends on the degree of precision required.

The instrument may be made, like the perspectometer, by drawing it on paper on a large scale, photographing and making a transparency which is bleached in bichloride of mercury.

138. PRECISION OF THE METHOD OF PHOTOGRAPHIC SURVEYING.— The precision of a survey executed by the methods exposed, when all the points are established by intersections, is the same as that of a plan plotted with a very good protractor or made with the plane table. There is, however, this difference: the number of points plotted by photography is greater than by the other methods.

Points plotted by means of their altitude below the station are far less accurate, their positions being given by the intersection of the visual ray with the ground plane, the angle of intersection being equal to the angle with the horizon plane or to the angle of depression of the point. With the camera employed, embracing 60°, this angle is always less than 30°; even that is seldom obtained in practice, a declivity of 30° being almost a precipice. Therefore, the intersection is always a poor one and the uncertainty becomes considerable with points near the horizon.

With perspective instruments, doing mechanically the same construction, the results are still less precise, being affected by the instrumental errors.

On the other hand, it must not be forgotten that when these methods are employed, the ordinary topographer would fall back on sketching; the results furnished by photography are therefore infinitely more precise.

The plan given at the end of this book and the photographs which accompany it, are specimens of actual work on the Topographical Survey.

CHAPTER IX.

PHOTOGRAPHS ON INCLINED PLATES.

139. GENERAL REMARKS.—Hitherto it has been assumed that the photographs used for the survey were taken on plates perfectly vertical. There are several cases in which this condition cannot be fulfilled: the camera may be an ordinary one, without any means of adjusting the plate, or the photographs may have been taken merely as illustrations, their employment for the construction of the plan being decided upon afterwards.

There are two classes of surveys in which the plates are always inclined. The first are secret surveys, the views being taken with a camera concealed about the person or otherwise. The scope of these surveys is very limited; the photographs, being instantaneous, lack detail in the distance, and, unless objects present great contrasts of light and shade, their images are somewhat indistinct as soon as the distance attains a few hundred yards. Another cause of trouble is the small size of the camera and plates: the views, being instantaneous, stand very little enlargement and the measurements are in consequence not very accurate.

The other class of surveys comprises those made from balloons. It is very doubtful whether the method will ever be found practical and prove of more than theoretical interest. It requires the consideration of an entirely new system of survey by means of photographs taken on plates placed horizontally or nearly so.

140. PLOTTING THE DIRECTIONS OF POINTS OF THE PHOTOGRAPHS.—When the photographic plate is not vertical, the picture plane of the perspective, which is parallel to the plate, is pierced by the vertical of the station. This trace is the vanishing point of all the vertical lines, which, having ceased to be front lines, are no longer represented by parallels to themselves.

Let $ABCD$ (Fig. 245), be a photograph on an inclined plate, P being the principal point and HH' the horizon line. The perpendicular $V\pi$ drawn through the principal point to the horizon line, is the principal line.

Revolve the principal plane on the picture plane around the principal line as an axis: the station falls in S_1 on a perpendicular PS_1 to VP, PS_1 being equal to the focal length.

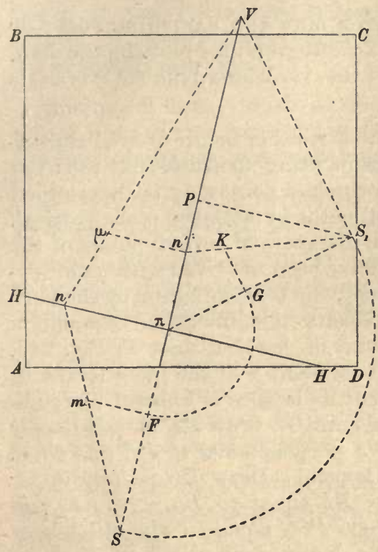


Fig. 245

Join $S_1\pi$ and S_1V; the first line is the revolved horizontal line from the station to the picture plane; S_1V is the revolved vertical of the station, and V the vanishing point of vertical lines.

Now revolve the horizon plane on the picture plane around the horizon line. The station comes in S, on the principal line produced, at a distance πS equal to πS_1.

To find the horizontal direction of a point μ of the photograph, draw the perspective of its vertical line by joining μ to V. The intersection n with the horizon line is the perspective of the trace in the horizon plane of the vertical of the point and Sn is its direction.

Comparing this construction with the one for vertical plates, we see that the same methods may be employed provided π be used as principal point, πS_1 as focal length, and that every point of the photograph be first projected on the horizon line by joining it to V, before measuring its distance from the principal line. The points such as n can be marked on a band of paper and used as was done for vertical plates.

With a plate nearly vertical, V is at a great distance from P, and the perspectives of the vertical lines have to be drawn with the centrolinead.

141. Determination of heights.—Let m, Fig. 245 be, on the ground plan, the point seen at μ on the photograph. Project on the principal plane the triangle formed by the visual ray, its projection on the horizon and the line $n\mu$. On the revolved principal plane, the projection of the visual ray is S_1n', $\mu n'$ being perpendicular to $V\pi$. The projection of m is F; it is revolved to G and the perpendicular GK to $S_1\pi$ is the projection of the vertical of the point or its height above the horizon plane.

Various devices may be imagined for constructing expeditiously the heights of a number of points.

142. Determination of the horizon line and vanishing point of verticals.—In order to make use of a photograph for plotting the plan, the horizon and principal lines and the vanishing point of verticals must be marked on the photograph.

It is assumed that the camera is available, either before or after the survey, for experimenting upon and that the focal length and principal point may be determined by the usual methods, with the plate vertical. If the zenith distances of several points of the photograph have been observed with a surveying instrument, the determination of the horizon line presents no difficulty. Assume a vanishing point of vertical lines V, Fig. 246, and join it to a point μ of the photograph of which the zenith distance is known. Through the principal point P, draw PE perpendicular to $V\mu$, and PS perpendicular to PE and equal to the focal length. Draw EG perpendicular to ES, take EF equal to $E\mu$, join SF and make the angle FSG equal to the altitude of μ; FG is the distance measured on $V\mu$ from μ to the horizon line.

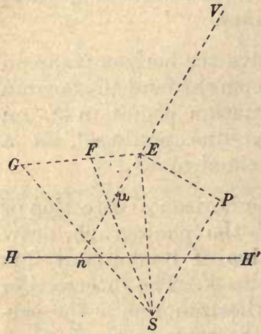


Fig. 246

Making μn equal to FG fixes one point n of the horizon line. A similar construction repeated on another point of the photograph furnishes a second point of the horizon line.

The first result will generally be inaccurate, because the position of the vanishing point V is only approximate. A new vanishing point must, therefore, be fixed by means of the horizon line just obtained, and the construction explained above is repeated. The second horizon line found will likely be sufficiently precise; if not, the construction must be made a third time.

In secret surveys, measured angles are seldom available, but it is easy to devise an attachment like a hand level, to mark the horizon line on the plate when the photograph is taken.

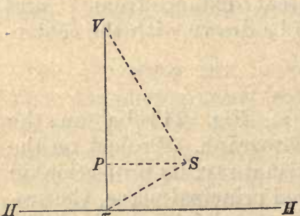


Fig. 247

Failing this, the horizon line must be furnished by the subject. When the view includes buildings, the vanishing point of

verticals is given by producing to their intersection the vertical lines of the buildings. This point V, Fig. 247, is joined to the principal point, P, and PS is made perpendicular to VP and equal to the focal length. Drawing $S\pi$ perpendicular to SV, the perpendicular HH to $V\pi$ is the horizon line.

Horizontal lines vanish on the horizon line; therefore, if the horizontal lines of two faces of a building be produced to their intersection, the line joining the two vanishing points is the horizon line.

If two intersecting horizontal lines appear on the photograph as a straight line, the latter is the horizon line.

Although angles cannot be measured, it may be possible to ascertain the points of the view which are at the same altitude as the observer; these joined together give the horizon line.

143. TRANSFERRING THE PERSPECTIVE TO A VERTICAL PLANE.—Instead of using exact copies of the negatives for plotting the plan, the copies or enlargements can be made in such a way that the perspective is restored to a vertical plane.

Have a copying, or enlarging camera, OCD (Fig. 248), movable on a horizontal axis passing through the first nodal point O and parallel to the negative.

Make an experimental negative with the field camera, the plate being vertical; draw on it the horizon and principal lines, place it in the holder of the copying camera, and mark the points of the holder corresponding to the horizon and principal lines. After inserting the holder, the camera is moved until the plate CD is vertical, and fixed in that position. The screen AB is now adjusted at the proper distance, parallel to the plate, and the projected images of the horizon and principal lines are marked on it, in such a manner, that the marks will appear on the prints.

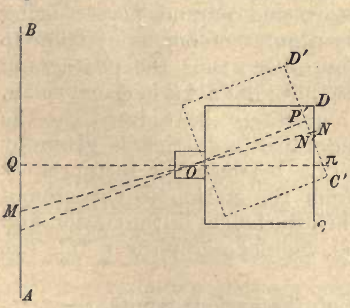


Fig. 248

To copy a negative taken in an inclined position, the horizon and principal lines are drawn on it, also a parallel to the horizon through the principal point. The negative is placed in the holder with the principal line on the proper marks, and the horizontal line of the principal point on the marks corresponding to the horizon line of the experimental plate. The camera is moved, up or down, until the image of the negative's horizon line π coincides with the horizon line Q previously marked on the screen; in this position

the image on the screen is the perspective restored to a vertical picture plane, because the inclination of the camera being the same as when the negative was taken, any point N' of the latter would have photographed in N on a vertical plate and given the same image M on the screen.

With a lens of sufficiently long focus and photographs taken nearly vertical, as is generally the case, the displacement of the camera is too small to affect the definition on the screen.

The holder must be provided with means for adjusting the negative; the principal point must always occupy the same position, the plate turning around it as a pivot.

The horizon and principal lines are indicated on the print by the marks fixed to the screen: the principal point has been displaced in copying and is now on the horizon line.

The change of picture plane can also be effected with the perspectograph, but the use of the instrument is not to be recommended when the change can be made so simply by photographic process.

144. PHOTOGRAPHS ON HORIZONTAL PLATES.—Photographs on horizontal plates might be obtained by an arrangement similar to the one described in § 100, with a pin-hole stop in the lens; they may be taken from a balloon with an ordinary camera, but the plates are only approximately horizontal.

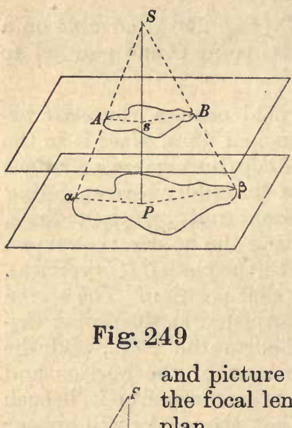


Fig. 249

The picture and ground planes being parallel, the figures of one are similar to those of the other; thus the photograph $\alpha\beta$ (Fig. 249) of a lake AB is also its plan, and only requires to be reduced to the proper scale. The reduction is given by the proportion between the distances Ss and SP from the station to the ground and picture planes. When the height of the station and the focal length are equal, the photograph is a full size plan.

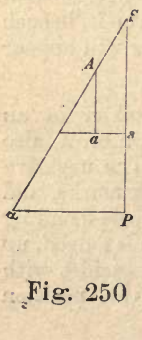


Fig. 250

To plot the directions of the various points, the principal point P of the photograph is placed on the foot of the station s, and a line of known direction, such as Pa, on the corresponding line of the plan sA. To find the direction of any other point B, its perspective β is joined to the principal point P: this line coincides with sB on the plan.

The height of a point is found by taking SP, Fig. 250, equal to the focal length and Ss equal to the height of

the station, drawing Pa and sa perpendicular to SP, Pa being equal to the distance of the point's perspective from the principal point, and sa equal to the distance on the plan from the station to the point. Join Sa; the parallel aA to SP is the height of the point above the ground plane.

A photograph taken from a balloon cannot be perfectly horizontal; to make use of it for plotting the plan, the trace s, Fig. 251, of the vertical of the station on the picture plane must be known.

The directions of the principal line sP and of the perpendicular to it, AB, are the same on the plan and on the photograph; they are different for all other lines.

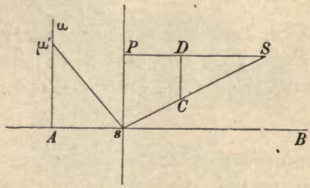


Fig. 251

To find the direction on the ground plan of a point μ of the perspective, draw PS perpendicular to the principal line and equal to the focal length, join Ss and take SC equal to the distance μA from μ to AB. Draw μA and CD parallel to Ps and take $A\mu'$ equal to SD; $s\mu'$ is the direction of the point on the ground plan. For $A\mu$ forms with its horizontal projection a right angle triangle in which the angle at A is the inclination of the plate to the horizon; this triangle is constructed in SCD, the angle S being the inclination of the plate. Therefore $A\mu'$, which is made equal to SD, is the horizontal projection of $A\mu$, μ' is the trace, on the ground plane, of the vertical of μ, hence the vertical plane passing through s and the point μ of the photograph cuts the ground plane along $s\mu'$.

A much better way to employ these photographs would be to restore them to a horizontal picture plane in printing, by the process of § 143, using Ps and AB in the same manner as the principal and horizon lines of the vertical photograph.

The great difficulty in balloon surveying is the determination of the trace of the vertical of the station on the picture plane, or of the foot of the station on the ground plan. The oscillations of the balloon prevent the use of any kind of level inside of the camera, and instrumental measurements of angles are open to the same objection. The angles might, however, be measured by two observers located on the ground.

In a view containing vertical lines, their vanishing point gives the trace of the vertical of the station; for a photograph taken from a short distance above buildings, this mode of determining the trace would be very convenient.

Balloon surveying appears adapted to military purposes only, although the advocates of the process are confident that it will eventually take the place of all other surveying methods.

SUPPLEMENTARY NOTE.

LAUSSEDAT'S NEW PHOTO-THEODOLITE.—Col. Laussedat has recently devised a light photo-theodolite shown in Figs. 252, 253 and 254 : the following description is given by the makers, Messrs. E. Ducretet & L. Lejeune, of Paris.

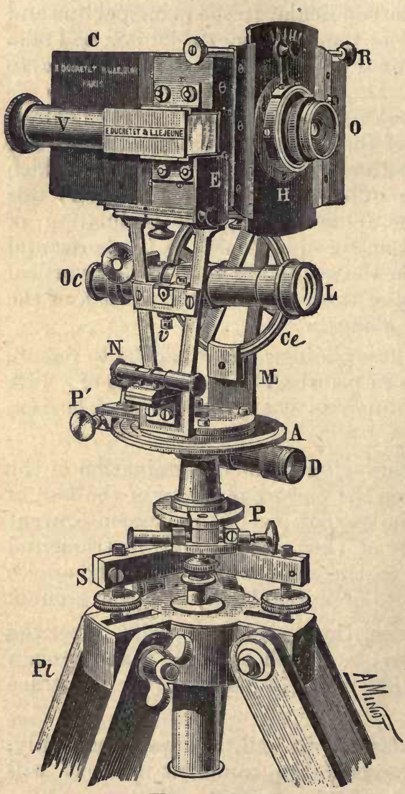


Fig. 252.

It consists of an ordinary surveying transit theodolite with a small camera on top.

Fig. 252 is the complete instrument on its stand, to which it is fixed by a central screw acting on the base S with foot screws. C is the camera, of very small size, with changing box for 15 plates $6\frac{1}{2}$x9 centimetres. With this box, plates are changed without removing the camera C from the instrument.

The changing box can be replaced by another one in full day light; the surveyor can carry with him several loaded boxes. Plates are subsequently enlarged to 18x24 centimetres or under.

The wide angle rectilinear lens O has a focal length of 75 millimetres; the angle between the horizontal points marked on the photographs is set at 60°. The lens is provided with an iris diaphragm and can be used with coloured screens, for obtaining better photographs of distances

and clouds. Focussing is rapidly effected by a special arrangement, with a graduation in metres on the plate H: the camera can thus be used for infinity, as in surveying, or for short distances as in photographing groups or other subjects.

The plate H carrying the objective, has a vertical motion: the camera must always be perfectly horizontal.

The finder V, with adjustment for focussing, shows the extent of the view covered by the sensitive plate. It is very bright and has a large aperture. A shutter E covers the object glass of this finder when not required.

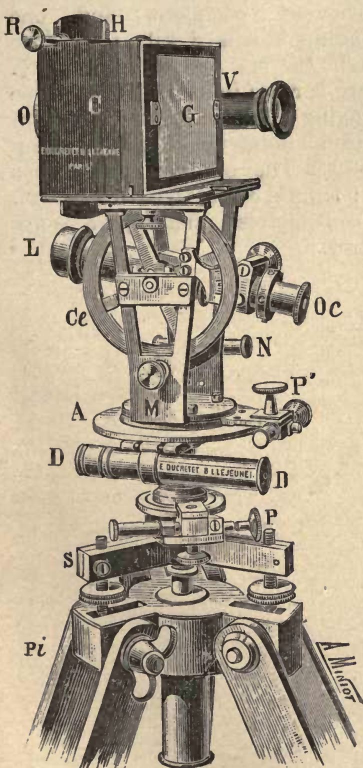


Fig. 253.

The telescope L, Oc, of the transit theodolite is provided with stadia wires. It is fixed to a vertical circle Ce, with vernier, clamp and slow motion screw, divided into half degrees: this graduation is made in grades when so ordered. The telescope L makes a complete revolution. It is placed midway between the uprights M in the vertical plane of the photographic objective O. This disposition secures the stability of the instrument, which is not realized when the telescope is at the side. N is the adjustable level. A is the horizontal circle with vernier divided into half degrees or grades. It is fixed by the clamp and slow motion screw P'.

D is a long compass with clamp and slow motion screw P, for reading, on circle A, the direction of the magnetic meridian. S is the base with foot screws fitting in the slits of the stand Pi.

Fig. 253 represents the same combination, the changing box C being removed and replaced by the ground glass.

Fig. 254 shows the camera C removed from the geodetic apparatus and set alone on the stand Pi by means of the additional spindle S'.

In this case, the whole camera and the geodetic apparatus are disconnected and can be employed separately.

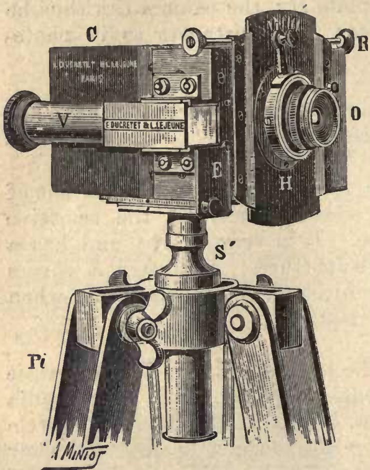
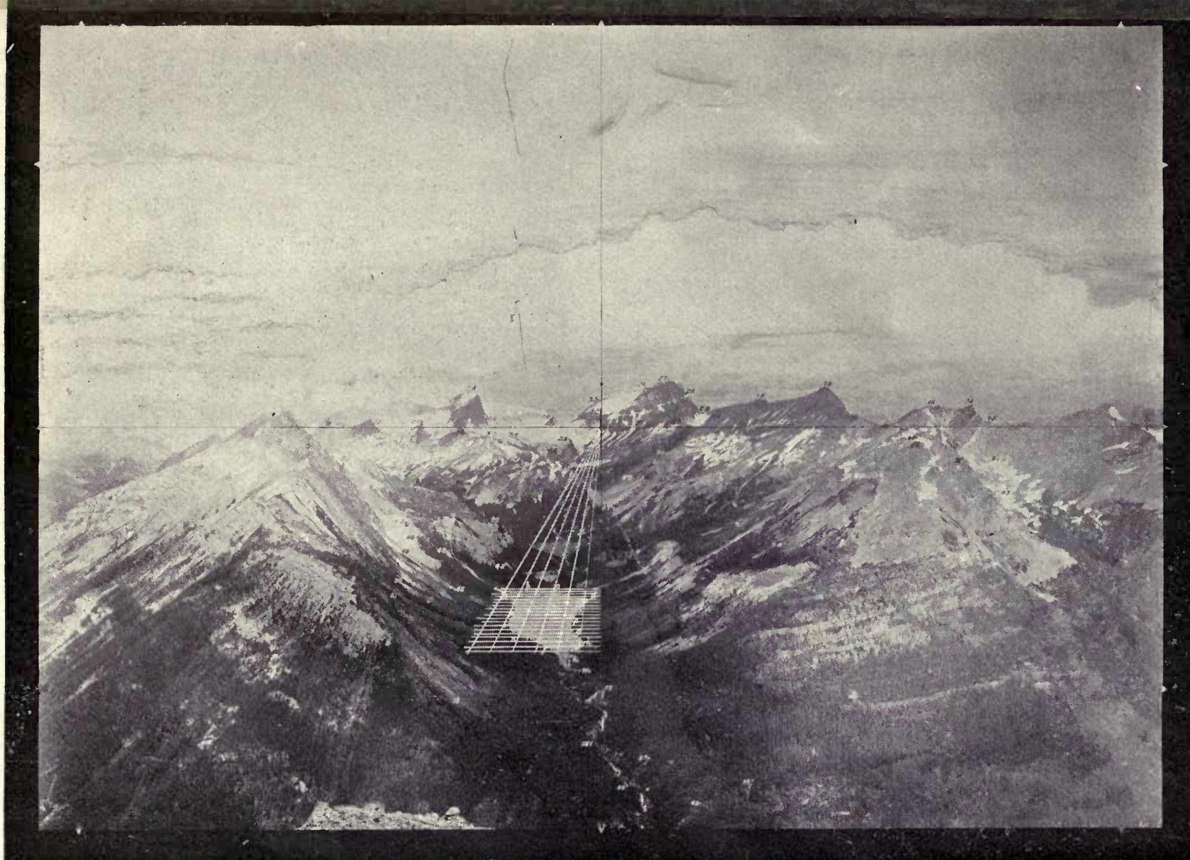


Fig. 254.

A carrying case, with shoulder straps, contains the whole instrument, including two changing boxes, each with 15 plates $6\frac{1}{2}$x9 centimetres. The weight of the case, instrument complete, and one changing box is 8 kil. 100. The stand *Pi* is carried separately. There is room in the case for three changing boxes, one artificial horizon, note books and reading glasses. The external dimensions are : length, 39 centimetres ; width, 17 centimetres ; height, 28 centimetres.

View No. 1—From BOUNDARY STATION.



View No. 2—From C. HECTOR STATION.

View No. 3—From NORTH FORK STATION.

View No. 4—From 2 HECTOR STATION,